MASS DISTRIBUTION OF SHOWER METEORS

by

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Progress Report No. 1 (May 1 - October 31, 1966).

NASA Grant NGR-52-042-003

THE UNIVERSITY OF ADELAIDE

DEPARTMENT OF PHYSICS

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1. INTRODUCTION

It has been shown (Elford, 1965) that the diurnal variation in hourly rate of radar echoes from a meteor shower depends on the distribution of the number of meteors as a function of mass. If the parameters of the radar system are known with sufficient precision the theoretical echo rate for a point source radiant of unit strength can be determined for all positions of the radiant in elevation and azimith. The result of such a calculation is a description of the response of the system to a radiant in any position in the sky and is appropriately called the 'response function' of the system. The form of the response function will vary according to the particular choice of the mass distribution. Hence a comparison of the observed and predicted radar echo rates for a particular meteor shower can lead to a determination of the mass distribution.

The procedure outlined above is being applied to the results of precise meteor echo rates measured at Christchurch (43.5°S) and Ottawa (45°N). The success of the analysis depends on accurately matching observed and predicted echo-rate versus time curves. In the first part of this project emphasis has been given to determining theoretical echo rate v. time plots for various mass distributions and for a range of values of declination of a point radiant. For this purpose the response functions of the radar systems at Christchurch and Ottawa have been recalculated in order to take into account overdense echoes and the range interval used in recording the radar data. These calculations are described in Section 2.

In order to reduce the number of graphs of the diurnal variation in echo rate a method of plotting contours of equal echo rate on a declination-time plot has been developed. The results are described in Section 3. The effect of a change in the mass distribution law on the rate plots is also discussed in Section 3.

The published data on radar echo rates determined at Christchurch give the total echo rate (shower plus sporadic) at any hour, and in order to determine the shower echo rate alone it is necessary to remove the sporadic background. This procedure is discussed in Section 4.

2. RESPONSE FUNCTIONS

The response functions for the Ottawa and Christchurch systems have been calculated according to the method developed by Elford (1964), but modified to include overdense trails, any form of the distribution of ionization along a trail and any range interval. The calculations have been carried out with the aid of a CDC 6400 computer and the program is given in Appendix I. For the purpose of machine calculation it was found necessary to smooth the polar diagram of the Christchurch antenna. The smoothed polar diagram data are given in Appendix II. The polar diagram for the Ottawa antenna was taken as that for crossed half-wave dipoles 0.4 wavelengths above a reflecting screen. The theoretical expression is given in Appendix II.

The incident flux of meteors whose radiants lie within an element of the celestial sphere of solid angle $d\Omega$ and which produce zenithal line densities greater than q_z can be written as $N(q_z)d\Omega$. This is the flux across a plane normal to the meteor paths. It is assumed that the cumulative distribution function $N(q_z)$ can be represented by a simple power law of the form $N(q_z) = Kq_z^{\ c}$. Typical response functions for the Ottawa and Christchurch systems are given in Figures 1 and 2 for several values of the exponent c. It can be seen that the radiant elevation at which these systems have their peak response depends on the value of c. In the case of the Christchurch system the minor low angle lobe in the antenna polar diagram produces a very marked variation of the position of the peak response as the exponent c is changed.

The Ottawa system records meteor echoes out to a maximum range of 360 km and it is of interest to examine the effect of this range restriction on the response function. The result is shown in Figure 3. It is clear that a reduction of the maximum range from 1,000 to 360 km has only a marginal effect on the response function for the Ottawa system. The effect of a similar range restriction on the Christchurch system has also been examined and the result is shown in Figure 4. In this case the restriction in range would have had a significant effect on the response function for radiant elevations in excess of 55°. The actual rate data obtained at Christchurch had no range restriction.

3. DIURNAL VARIATION IN ECHO RATE

As a radiant moves across the sky the elevation, θ, from a given station can be determined from the following expression

$$\sin\theta = \cos\frac{\pi}{12}t \ \cos\delta \ \cos L + \sin\delta \ \sin L,$$

where L is the latitude of the station, δ is the declination of the radiant and t is the time in hours after transit. Hence for a given station and a given radiant the variation in the theoretical echo rate as a function of time can be determined by calculating the radiant elevation at any given time and noting the value of the response function at this elevation.

The theoretical echo rate of meteors from point radiants as a function of time has been computed for the Ottawa and Christchurch systems. The calculations were carried out at intervals of 5 degrees in radiant declination and for five values of the exponent c in the mass distribution law, viz. - 0.6, - 0.8, - 1.0, - 1.2, - 1.4. For convenience the results are presented as contours of equal rate on a declination-time plot. The results for the Ottawa system are presented in Figures 5 - 9 and for the Christchurch system in Figures 10 - 14. The contours range from zero to a maximum rate of ten units. The approximate positions of the contours of zero rate are indicated by dashes and the positions of the contours of maximum rate by plus symbols.

The rate contour diagrams for the Ottawa system show that radiants with declinations higher than + 10° give a rate-time variation with two maxima spaced equally either side of the time of transit. For radiants with declinations between + 35° and + 55° the echo rate goes to zero for about an hour before and an hour after transit. The main effect of a change in the value of c from - 0.6 to - 1.4 is to expand the region occupied by the echo rate 'well'. Hence at declinations higher than + 10° the time of occurrence of the peak rate can change by as much as one hour when c is changed from - 0.6 to - 1.4.

The general features of the rate contour diagrams for the system at Christchurch are similar to those for the Ottawa system but the effect of a change in the value of c is more marked. For c = - 0.6 (Figure 10) a reduction in echo rate near transit only occurs for radiants lying between declinations - 25° and - 60° and the maximum duration of the decrease in echo rate is three hours. In Figure 14 the contours of constant echo rate are drawn for c = - 1.4 and from this diagram it can be seen that all radiants south of declination + 5° give rate time diagrams with a minimum at the time of transit and that for radiants lying between declinations 0 and - 75° the duration of the decrease in echo rate ranges from 4 to 14 hours.

Typical cross-sections of an echo rate contour diagram are given in Figure 15 where rate-time curves are plotted for the Christchurch system at intervals of 5° in radiant declination and for c = - 1.0. These curves show that for a radiant with a medium southern declination the observed echo rate as a function of time can exhibit quite complicated behaviour.

We have already seen that the whole iso-rate diagram is altered by a change in the mass distribution law. A close examination shows that the effect of these changes is most marked for radiants whose zenith distance at transit is less than 30°. The extent of the dependence of the echo rate on the mass distribution law is illustrated in Figures 16 and 17 where normalized diurnal rate curves for the Ottawa and Christchurch systems are plotted for three values of c and for two values of declination. In the case of the Ottawa system the diurnal rate curves tend to become narrower as the value of c becomes more negative. For the Christchurch system changes in the values of c cause only marginal changes in the form of the rate curves for low declination radiants. However for radiants at high declination a change of c from - 0.6 to - 1.4 completely alters the form of the diurnal rate curve and even a change of 20% in c produces significant changes in the rate-time plot.

4. OBSERVED DIURNAL RATE VARIATIONS

Detailed accounts have been published of radar echo counts of meteors carried out at Ottawa and Christchurch. In order to determine the diurnal variation in shower activity an estimate of the sporadic activity must be subtracted from the total echo rate. This is being carried out in the following way. The published data have been re-plotted in terms of the day-to-day variation in total echo rate during one year for each hour of local time. The times of occurrence of known showers are marked on the annual plots and a base rate is drawn by eye. Any inconsistencies in successive hourly plots are noted and appropriate corrections made. The base rate is subtracted from the total echo rate and the diurnal shower activity plotted for several days near the peak of each shower. The chief uncertainties occur when showers overlap in time and in general it is not possible to deduce diurnal rate curves in these cases.

Results of radar echo counts of Geminid meteors observed with the Ottawa system have been published by McIntosh (1966). The rates of the short duration echoes as a function of radiant elevation angle were compared with that predicted using c = - 0.5 and - 2.0. Neither of these rate curves fit the observations. The width of the observed curves is indicative of a value of c close to - 2.0 but the position of the peak suggests c = - 0.5. Thus a simple inverse power mass distribution is not adequate to describe the observed diurnal rate variations of Geminid echoes. A better approximation would be obtained by replacing the single power term by the first three terms of a power series whose coefficients are to be determined by the comparison of predicted and observed rate curves. A program to give the appropriate response functions is being written.

APPENDIX I COMPUTER PROGRAMS

```
      PROGRAM F101 (INPUT,TAPE60=INPUT,OUTPUT,TAPE61=OUTPUT,PUNCH)
C     CALCULATION OF RESPONSE FUNCTION
C     FOR OTTAWA AND CHRISTCHURCH SYSTEMS
      DIMENSIONHEAD(36),JELEVN(91),GAIN(92),RESPON(91),HTEXT(12),TEXTNM(
     112)
      COMMONHTEXT,TEXTNM
C     FUNCTION DEFINITION
      CSPHIF(HITE,RANGE)=(HITE/RANGE-RANGE/(2.*RE))/CSTHR
C     INITIALIZE
      RE=6.37E+3
      RO=1000.*RE
      D=57.296
      READ3,THMIN,DECTH,THMAX
      N=(THMAX-THMIN)/DECTH+1.001
      NANT=N
      READ(60,2)(GAIN(J),J=1,N)
    2 FORMAT(10F6.1)
C     NORMALIZE POLAR DIAGRAM
C     INTEGRAL OVER ELEVATION ALONE MUST COME TO 2.0  .
      SUM=0.
      DTHETA=DECTH/D
      NN=N-1
      DO110I=1,NN
      CSFL=COSF((FLOATF(I)-0.5)*DTHETA)
      S=GAIN(I)+GAIN(I+1)
      SUM=SUM+S*CSFL
  110 CONTINUE
      HLFINT=SUM*DTHETA/4.
      DO111I=1,N
  111 GAIN(I)=GAIN(I)/HLFINT
      GAIN(N+1)=GAIN(N-1)
      READ3,WAVELN,POWR,SIGMIN
      READ3,STALAT,STLONG,ZNLONG
    3 FORMAT(7E10.2)
    4 READ3,THRMIN,DECTHR,THRMAX
      IF(THRMIN)204,6,6
    6 N=(THRMAX-THRMIN)/DECTHR+1.0001
      KWPOWR=  POWR
      TRPOWR=  POWR*1000.0
      CSENS=SQRTF(3)17.*SIGMIN/(WAVELN**3*TRPOWR))
      READ3,DECPH
      READ(60,11)(HEAD(I),I=1,36)
   11 FORMAT(12A6)
    8 READ3,RMIN,RMAX
C     SETTING  FLUX  NEGATIVE MAKES S/R NUMDIS READ IN  C  AND  K
      FLUX=-1.
      IF(RMIN)4,10,10
C     COMPUTE RESPONSE AT EACH DESIRED RADIANT ELEVATION  THR
   10 THR=THRMIN+.00001
      DO23J=1,N
      JELEVN(J)=THR
      SUMH=0.
      MORE=0
   16 CSTHR=COSF(THR/D)
      SNTHR=SINF(THR/D)
C     INTEGRATE WITH RESPECT TO HEIGHT
   24 CALLHTDIS(H,WDH,MORE)
      SUMPHI=0.
C     FIND MAXIMUM VALUE OF AZIMUTH ANGLE PHI IN ECHO PLANE
C     CSPHIF  IMPLEMENTS EQUATIONS  (5) AND (6) OF HRMP NASA REPORT 8
  213 CSPH=CSPHIF(H,RMAX)
      IF(CSPH-1.)12,12,21
C     IF PHIMAX DOES NOT EXIST, THE MEAN HEIGHT OF THIS INTERVAL IS ABOVE
C     THE RMAX CIRCLE IN THE ECHO PLANE FOR THIS VALUE OF  THR
C     HENCE NO CONTRIBUTION
C     IF  PHIMAX  EXISTS PROCEED SIMILARLY WITH  PHIMIN
C     IF PHIMIN DOES NOT EXIST SET IT TO ZERO
   12 PHIMAX=ACOS(CSPH)*D
  133 CSPH=CSPHIF(H,RMIN)
      IF(CSPH-1.)14,14,13
   13 PHIMIN=0.$  GOTO15
   14 PHIMIN=ACOS(CSPH)*D
   15 L=(PHIMAX-PHIMIN)/DECPH+1.
C     INTEGRATE FROM PHIMAX TO PHIMIN, WITH A RESIDUAL ELEMENT ADJACENT
C     TO PHIMIN INSTEAD OF ONE WITH WIDTH  DECPH
 1003 DO20KK=1,L
      PHIB=PHIMAX-DECPH*FLOATF(KK)
      PHIA=PHIB+DECPH
      PHIB=MAX1F(PHIB,PHIMIN)
      DPHI= PHIA-PHIB
      PHI=(PHIA+PHIB)/2.
      SNTH=CSTHR*COSF(PHI/D)
      X=SQRTF(SNTH**2+2.0*H/RE)
      RANGP=RO*(X-SNTH)
      DSBYDH=RANGE/X*DPHI/D
      POSNJ=(ASIN(SNTH)*D-THMIN)/DECTH+1.001
      JP=POSNJ
      JP=JP+XDIMP(1,JP)-XDIMP(JP,NANT)
      POSDEC=POSNJ-FLOATF(JP)
      G=GAIN(JP)*(1.-POSDEC)
      G=G+GAIN(JP+1)*POSDEC+.001
      REFCO=CSENS*RANGP**1.5/G
C     REFCO IS FROM KAISER AND CLOSS  EQ(27)
      IF(REFCO-0.66)30,30,31
   30 QD=1.13E14*REFCO
C     ABOVE UNDERDENSE VALUE OF QD FROM KAISER AND CLOSS  EQ(25)
      GOTO32
   31 QD=3.906E14*REFCO**4
   32 QZ=QD/SNTHR
      CALLNUMDIS(QZ,FLUX)
   20 SUMPHI=SUMPHI+FLUX*DSBYDH
   21 SUMH=SUMH+SUMPHI*WDH
      IF(MORE)24,22
   22 RESPON(J)=SUMH*2.
C     THE FACTOR 2.0 IS FOR THE EQUAL CONTRIBUTION FROM NEGATIVE VALUES OF PHI
      THR=THR+DECTHR
   23 CONTINUE
C     OUTPUT
      TH1MIN=THMIN
      TH2MIN=THMIN
      TH1MAX=THMAX
      TH2MAX=THMAX
      DECTH1=DECTH
      DECTH2=DECTH
      PHSITE=0.
      RSITE=0.
      PUNCH100,STALAT,STLONG,ZNLONG,RMIN,RMAX
      PUNCH100,RESPON
  100 FORMAT(6E12.3)
      PUNCH11,HPAD
      PUNCH101,HTEXT
      PUNCH101,TEXTNM
  101 FORMAT(6A10)
      WRITE(61,300)(HEAD(I),I=1,36)
  300 FORMAT(A6,24X,10A6,24X,A6)
```

```
      PRINT301,KWPOWR,RMIN,RMAX,WAVELN,SIGMIN
  301 FORMAT(1H ,52X,10HPARAMETERS// 17X,19HTRANSMITTING SYSTEM,49X,16HR
     1ECEIVING SYSTEM//10X,10HPEAK POWER,I5,3H KW,49X,12HRANGE LIMITS F6
     1.1,1H,F6.1,4H KMS/10X,11HWAVE LENGTH,F5.1,7H METERS,44X,25HMINIMUM
     1 DETECTABLE SIGNAL  E10.2,6H WATTS )
      PRINT302,TH1MIN,DECTH1,TH1MAX,TH2MIN,DECTH2,TH2MAX
  302 FORMAT(10X28HANTENNA ELEVATION PARAMETERS,F5.1,1H,,F5.1,1H,,F5.1,5
     1H DEG.,17X28HANTENNA ELEVATION PARAMETERS F5.1,1H,F5.1,1H,F5.1,5H
     1DEG.  //)
      PRINT303,THRMIN,DECTHR,THRMAX
  303 FORMAT(40X28HRADIANT ELEVATION PARAMETERS,F5.1,1H,,F5.1,1H,,F5.1,5
     1H DEG.//)
      PRINT304,TEXTNM
  304 FORMAT(40X19HFLUX LAW PARAMETERS /(44X3A10))
      PRINT305,DECPH,HTEXT
  305 FORMAT(// 40X25HINTEGRATION IN ECHO PLANE /44X20HAZIMUTH ANGLE STE
     1P = F5.1,5H DEG. ///40X19HHEIGHT DISTRIBUTION /(44X3A10))
      PRINT201,(JELEVN(J),GAIN(J),J=2,NANT)
  201 FORMAT(// 15X 90HANTENNA GAIN RELATIVE TO AN ISOTROPIC RADIATOR AS
     1 A FUNCTION OF REFLECTION POINT ELEVATION / (10(I7,F6.2)))
      PRINT202,(JELEVN(J),RESPON(J),J=2,N)
  202 FORMAT( //34X43HRESPONSE AS A FUNCTION OF RADIANT ELEVATION / (10(
     1I4,E9.2)))
      GOTO8
  204 STOP
      END
```

```
      SUBROUTINE NUMDIS(OZ,FLUX)
      DIMENSIONTEXTNM(12),HTEXT(12)
      COMMONHTEXT,TEXTNM
      IF(FLUX)1,4,4
C     A NEGATIVE VALUE FOR  FLUX  CAUSES NEW VALUES OF  C  AND  K  TO BE READ
    1 READ2,VALUEK,C
    2 FORMAT(2E10.2)
      ENCODE(120,3,TEXTNM)VALUEK,C
    3 FORMAT(3HK= F10.2,17X3HC= F5.2)
    4 FLUX=VALUEK*OZ**C
      RETURN
      END
```

```
      SUBROUTINE HTDIS(H,WDH,MORE)
      DIMENSIONHTEXT(12),TEXTNM(12),WTABLE(20)
      COMMONHTEXT,TEXTNM
      DATA((WTABLE(I),I=1,20)=20*1.0)
      DATA(HMIN=90.)
      DATA(NMAX=13)
      HSTEP=0.5
      IF(MORE)3,1
    1 MORE=1
      NSTEP=0
      ENCODE(120,2,HTEXT)HSTEP
    2 FORMAT(30HRECTANGULAR                   ,30HBOUNDARIES 90-96 KM
     1            ,1AHINTEGRATION STEP  F5.2,4H KM     )
    3 H=HMIN+HSTEP*FLOATF(NSTEP)
      NSTEP=NSTEP+1
      WDH=WTABLE(NSTEP)*HSTEP*1000.
      IF(NSTEP-NMAX)5,4,4
    4 MORE=0
    5 RETURN
      END
```

```
      PROGRAM PRRATE(INPUT,TAPE60=INPUT,OUTPUT,TAPE61=OUTPUT,PLOT,TAPE1=
     1PLOT)
C     DIURNAL RATE VARIATION AS A FUNCTION OF RADIANT DECLINATION,
C     GIVEN STATION LATITUDE AND RESPONSE FUNCTION.
C     THIS VERSION FOR RESPONSE FUNCTIONS WHICH ARE INDEPENDENT OF AZIMUTH.
C     RESPONSE MUST BE SPECIFIED EACH 1 DEGREE OF ELEVATION.
C     PLOTS AND PRINTS PREDICTED RATES CENTRALLY IN TIME ABOUT TRANSIT.
      DIMENSIONRESPON(92),HEAD(36),RATE(73,37),MRATE(24),TIME(73,2),ITIM
     1E(25),MRASUM(37),LQ(73,37),CO(12),HTEXT(3,4),TEXTNM(3,4)
      READ(60,3)STALAT,STLONG,ZNLONG,RMIN,RMAX
      READ(60,3)(RESPON(J),J=1,91)
    3 FORMAT(6E12.3)
      RESPON(92)=RESPON(90)
      XSCALE=10.
      YSCALE=5.
      DEG=57.296
      RESMAX=0.
      DO5J=1,91
    5 RESMAX=MAX1F(RESMAX,RESPON(J))
      FACTOR=RESMAX/1000.
   10 READ(60        ,1)(HEAD(I),I=1,36)
    1 FORMAT(12A6)
   11 WRITE(61,101)HEAD
  101 FORMAT(A6,24X,10A6,24X,A6)
      READ102,HTEXT
      READ102,TEXTNM
  102 FORMAT(6A10)
      PRINT103,((TEXTNM(I,J),I=1,3),(HTEXT(I,J),I=1,3),J=1,4)
  103 FORMAT(1H010X19HFLUX LAW PARAMETERS  60X19HHEIGHT DISTRIBUTION  //
     1(15X3A10,50X3A10))
      PRINT104,RMIN,RMAX
  104 FORMAT(   50X12HRANGE LIMITS  F6.0,3H -  F6.0,3H KM  )
      SNLAT=SINF(STALAT/DEG)
      CSLAT=COSF(STALAT/DEG)
      LAT=XFIXF(STALAT)/5
      MRMAX=0
C     FIND RADIANT DECLINATION LIMITS
C     STORE ZERO RATE VALUE FOR ALL RADIANTS OUTSIDE THEM.
      MINDEC=XMAX0F(0,LAT)+1
      IF(MINDEC-1)502,502,500
  500 LIM=MINDEC-1
      DO501IDEC=1,LIM
      DO501JTIME=1,73
  501 RATE(JTIME)=0.
  502 MAXDEC=XMIN0F(0,LAT)+37
      IF(MAXDEC-37)503,505,505
  503 LIM=MAXDEC+1
      DO504IDEC=LIM,37
      DO504JTIME=1,73
  504 RATE(JTIME,IDEC)=0.
      ND=MAXDEC-MINDEC+1
C     TABULATED DECLINATIONS WILL BE AT 5 DEGREE INTERVALS
C
C     PRINT COLUMN HEADINGS
  505 DO300I=1,25
  300 ITIME(I)=I-13
      WRITE(61,301)(ITIME(I),I=1,24)
  301 FORMAT(///26X76HTIME VARIATION OF NORMALIZED ECHO RATE AS A FUNCTI
     1ON OF RADIANT DECLINATION  ///48X36HTIME IN HOURS AFTER RADIANT TR
     2ANSIT    /12X24I5/12H RADIANT DEC   )
C     GENERATE TIME VECTORS FOR PLOTTING
      DO302I=1,73
      TIME(I,1)=8.6-FLOATF(I-1)/10.
  302 TIME(74-I,2)=TIME(I,1)
C     CALCULATE RATES
      DO350IDEC=MINDEC,MAXDEC
      MRASUM(IDEC)=0
      JDEC=5*(IDEC-1)-90
      CSDEC=COSF(FLOATF(JDEC)/DEG)
      SNDEC=SINF(FLOATF(JDEC)/DEG)
      DO340IHOUR=1,24
      DO330MIN20=1,3
      JTIME=3*(IHOUR-1)+MIN20
      HANEG=-15*ITIME(IHOUR)-5*(MIN20-1)
      DL2=COSF(HANEG/DEG)*CSDEC
      DN2=SNDEC
      DN1=DL2*CSLAT+DN2*SNLAT
      IF(DN1)179,179,180
  179 RATE(JTIME,IDEC)=0.
      GOTO330
  180 CSELEV=SQRTF(1.0-DN1**2)
      ELEVN= DEG*ATANF(DN1/CSELEV)
      IF(ELEVN-89.99)181,179,179
  181 J=ELEVN+1.
      ELDEC=MODF(ELEVN,1.)
C     QUADRATIC INTERPOLATION
      DELR=RESPON(J+1)-RESPON(J)
      DELR2=RESPON(J+2)-RESPON(J+1)
      DELSQR=DELR2-DELR
      RATE(JTIME,IDEC)=(RESPON(J)+ELDEC*(DELR+(ELDEC-1.)*DELSQR/2.))/FAC
     1TOR
  330 CONTINUE
      MRATE(IHOUR)=RATE(JTIME-2,IDEC)+0.5
      MRASUM(IDEC)=MRASUM(IDEC)+MRATE(IHOUR)
  340 MRMAX=XMAX0F(MRATE(IHOUR),MRMAX)
C     PRINT RATES
      WRITE(61,341)JDEC,MRATE
  341 FORMAT(4XI3,5X24I5)
  350 RATE(73,IDEC)=RATE(1,IDEC)
      PRINT303,RESMAX
  303 FORMAT(/10X22HACTUAL MAXIMUM RATE =   E10.2)
C     DETERMINE WHICH FORM(S) OF PLOTTING ARE REQUIRED
C       IPLOT = 0  STOP
C               1  HOURLY PLOTS
C               2  CONTOUR PLOTS
C               3  BOTH
      READ(60,401)IPLOT
  401 FORMAT(I1)
      IF(IPLOT-1)402,403,403
  403 CALLPLOTS(5HSPENS,9,1)
      RBASE=4.
      IF(IPLOT-2)404,405,404
C     HOURLY PLOTS
  404 RNORM=MRMAX/4
C     WRITE AXIS LABEL
      CALLSYMBOL(3.4,6.1,.14,19HHOURS AFTER TRANSIT,270.,19)
      DO400IDEC=MINDEC,MAXDEC
      LINE=XMODF(IDEC+1,2)+1
      JDEC=5*(IDEC-1)-90
      GOTO(360,361),LINE
C     WRITE DECLINATION
  360 CALLNUMBER(RBASE+0.5,9.9,.14,JDEC,270.,10H6HDEC = I3)
C     PLOT AXIS WITH TICKS, AND VALUES IF THIS IS FIRST GRAPH
      DO356I=1,25
      CALLSYMBOL(RBASE-.04,8.90-.3*FLOATF(I),.08,16,270.,-1)
```

```
      IF(IDEC-MINDEC)356,354,356
  354 IF(XMODF(I,2))356,356,355
  355 CALLNUMBER(3.7,9.14-.3*FLOATF(I),.14,ITIME(I),270.,2HI3)
  356 CONTINUE
      CALLPLOT(RBASE,1.0,3)
      CALLPLOT(RBASE,9.0,2)
      GOTO(361,400),LINE
C     NORMALIZE AND PLOT RATES, IN DIRECTION WHICH MINIMIZES PEN TRAVEL
  361 CALLPLOT(RBASE,TIME(1,LINE),3)
      IPEN=2
      DO390J=1,73
      JTIME=J+(LINE-1)*(74-2*J)
C     LIFT PEN IF RATE ZERO
      IF(RATE(JTIME,IDEC))3611,3611,3612
 3611 IPEN=3
      GOTO390
 3612 CALLPLOT(RATE(JTIME,IDEC)/RNORM+RBASE,    TIME(J    ,LINE),IPEN)
      IF(IPEN-2)364,363,362
  362 IPEN=2
      GOTO364
  363 IPEN=1
  364 IF(JTIME-37)390,365,390
C     INDICATE Y AXIS
  365 CALLPLOT(RBASE,5.0,1)
      CALLPLOT(RATE(JTIME,IDEC)/RNORM+RBASE,5.0,3)
      IPEN=2
  390 CONTINUE
      IF(IPEN-2)391,391,392
  391 CALLPLOT(RBASE,TIME(73,LINE),2)
  392 GOTO(400,360),LINE
C     SHIFT BASE LINE FOR NEXT GRAPH
C     TAKING ACCOUNT OF MEAN RATE IN THIS GRAPH
  400 RBASE=RBASE+MAXIF(2.,FLOATF(MRASUM(IDEC)/(MRMAX*6)+1))
C     LIFT PEN, AND TERMINATE PLOT OR PROCEED WITH CONTOUR PLOT
      XNEW=RBASE+2.
      YNEW=0.
  405 CALLPLOT(XNEW,YNEW,-3)
      IF(IPLOT-1)402,402,406
C     CONTOUR PLOT
  406 CALLPLOT(0.,1.,-3)
      CALLCNTOUR(RATE,73,ND,XSCALE,YSCALE,11,0,LQ,CO,NONO)
      CONINT=CO(2)-CO(1)
      PRINT407,NONO,CONINT,CO(1)
  407 FORMAT(//1XI3,32H CONTOURS DRAWN AT INTERVALS OF E10.3,
     113H STARTING AT E10.3)
      XNEW=0.
      YNEW=-1.
      IPLOT=0
      GOTO405
  402 CALLEXIT
      END
```

```
      SUBROUTINE CNTOUR(H,M,N,XSCALE,YSCALE,NO,IFCO,LQ,CO,NONO)
C
C     CSIRO 3600 LIBRARY ROUTINE LIN MODIFIED FOR 6400
C     CHOICE OF CONTOUR VALUES IMPROVED
C     FRAME DRAWN TO INDICATE PLOTTING AREA BOUNDARY.
C
C     H(I, J)     IS THE ARRAY OF POINTS FOR WHICH A CONTOUR MAP IS TO BE DRAWN
C     M,N         ARE THE DIMENSIONS OF THE ARRAY AT CALL TIME.
C                 M  MUST BE EQUAL TO  I , BUT  N  MAY BE SMALLER THAN  J ,
C                 IF DESIRED, RESULTING IN A SAVING OF CP TIME WHEN ONLY PART
C                 OF ARRAY  H  IS OF INTEREST.
C     NO          IS THE NUMBER OF CONTOURS(1.LE.NO.LE.30 )  ASKED FOR.
C     CO(NO)      ARE THE LEVELS OF THE CONTOURS IN ASCENDING ORDER. DIMENSION
C                 ( NO ). CO MAY BE SET AUTOMATICALLY OR PRESET.
C     NONO        IS THE NUMBER OF CONTOURS ACTUALLY DRAWN IF  CO  IS ..
C                 SET AUTOMATICALLY,  (NONO.LE.NO)  .
C     IFCO        IS AN INDICATOR.
C                 IF IFCO = 0 , THEN THE VALUES OF CO ARE SET AUTOMATICALLY
C                 AT SUCCESSIVE SIMPLE NUMBERS BETWEEN THE LOWEST AND THE
C                 HIGHEST VALUES IN  H  .
C                 (SIMPLE NUMBERS IN THIS CONTEXT ARE MEMBERS OF SOME SERIES
C                   (K1*10.**K2*SIMPLE)
C                 WHERE K1 IS AN INTEGER RUNNING INDEX,
C                         K2 IS AN INTEGER SET BY THE SUBROUTINE, AND
C                         SIMPLE IS A PLEASING INTEGRAL SUBMULTIPLE OF 10.0 ).
C                 IF IFCO=1 ,THE PRESET VALUES OF CO WILL BE USED.
C     LQ(I, J)    IS WORKING SPACE, DIMENSIONED AT LEAST (M,N) IN CALLING PROG.
C     XSCALE, YSCALE ARE SCALES IN CELLS/INCH OF M, N RESPECTIVELY.
C
C     THE PEN IS TAKEN TO BE AT THE LOWER LEFT CORNER OF THE  M X N  GRID,
C                 I.E. (1,1), AND THIS POINT MUST HAVE BEEN DEFINED AS ORIGIN
C                 BY A PREVIOUS  STATEMENT   CALL PLOT(X,Y,-3) .
C     AFTER ALL CONTOURS ARE DRAWN THIS S/R DEFINES A NEW ORIGIN FOR FURTHER
C                 PLOTTING, AT   ((M-1)/XSCALE+2., 0.)  .
C
      DIMENSIONH(M,N),LQ(M,N),CO(NO)
      DIMENSIONSIMPLE(5),JLIST(1)
      DIMENSIONLEQ(3),KSIDE(3)
      DIMENSIONINEW(22),JNEW(1),NIDLIST(1),KLIST(1),ILIST(1),KK(1),KL(1)
      EQUIVALENCE           (INEW(2),JNEW), (INEW(5),NIDLIST),
     *                (INEW(12),KLIST), (INEW(15),ILIST), (INEW(19),KK),
     *                (INEW(21),KL),(INEW(16),JLIST)
      DATA (INEW = 1, 0,-1,0, 1,2,3,4,1,2,3, 2,1,2, 1,0,0,0, 1,0, 1,-1)
      DATA(SIMPLE=1.,2.,2.5,5.,10.)
      DATA(LSTBIT=1B),(NOLAST=77777777777777777776B),(LEFT=7777777777000
     10000000B)
      LOGICALSTART,LOOKBK,G,LEQ,LQ,LSTBIT,NOLAST,WORD,RESULT,MASK,LEFT
C     MASKS AND FUNCTIONS FOR DEALING BIT-BY-BIT WITH LQ, LEQ.
      ASSIGN1000TONSET1
      ASSIGN1001TONSET0
      ASSIGN1002TONGET
      GOTO1004
 1000 MASK=LSTBIT
      CALLSHIFT(MASK,30*KBIT-LCO)
      RESULT=OR(WORD,MASK)
      GOTONCALL
 1001 MASK=NOLAST
      GOTO1003
 1002 MASK=LSTBIT
 1003 CALLSHIFT(MASK,30*KBIT-LCO)
      RESULT=AND(WORD,MASK)
      GOTONCALL
```

```
1004 CONTINUE
     XMAX=FLOATF(M-1)/XSCALE
     YMAX=FLOATF(N-1)/YSCALE
C    DRAW FRAME WITH TICKS
     CALLPLOT(0.,YMAX,3)
     CALLPLOT(0.,0.,2)
     CALLPLOT(XMAX,0.,1)
     CALLPLOT(XMAX,YMAX,1)
     CALLPLOT(0.,YMAX,1)
     NN=N-1
     DO90I=2,NN
  90 CALLSYMBOL(-.04,FLOATF(N-I)/YSCALE,.08,16,270.,-1)
     MM=M-2
     DO91I=1,MM
  91 CALLSYMBOL(FLOATF(I)/XSCALE,-.04,.08,16,0.,-1)
     XMAX=XMAX+.04
     YMAX=YMAX+.04
     NN=NN-1
     DO92I=1,NN
  92 CALLSYMBOL(XMAX,FLOATF(I)/YSCALE,.08,16,270.,-1)
     MM=MM+1
     DO93I=2,MM
  93 CALLSYMBOL(FLOATF(M-I)/XSCALE,YMAX,.08,16,0.,-1)
     DO 61  J = 1, N
     DO 61  I = 1, M
  61 LQ(I,J)=0
     IF(IFCO)52,56
  52 NONO=NO
     GOTO55
C    FIND RANGE OF FUNCTION H
  56 HMIN=1.E304
     HMAX = -HMIN
     DO 57  I = 1, M
     DO 57  J = 1, N
     HMIN = MIN 1 F (HMIN, H(I,J))
  57 HMAX = MAX 1 F (HMAX, H(I,J))
C    GENERATE SIMPLE CONTOUR INTERVAL
     SUBRNG=(HMAX-HMIN)/FLOATF(NO-1)
     RNGELG=LOG10F(SUBRNG)
     LGRNGE=RNGELG+SIGNF(0.5,RNGELG)-0.5
     CHAR=10.**LGRNGE
     DO70I=1,5
     ADD=SIMPLE(I)*CHAR
     IF(ADD-SUBRNG)70,71,71
  70 CONTINUE
C    SET CONTOUR VALUES
  71 COZERO=HMIN-MODF(HMIN,ADD)
     DO72NONO=1,NO
     CO(NONO)=COZERO+ADD*FLOATF(NONO-1)
     IF(CO(NONO)-HMAX)72,72,73
  72 CONTINUE
     NONO=NO
     GOTO55
  73 DO74I=NONO,NO
  74 CO(I)=0.
     NONO=NONO-1
C    CONSIDER THE BITS IN A WORD OF  LQ  TO BE NUMBERED FROM 1 AT LEFT
C    TO 60 AT RIGHT.
C    SET HIT LCO TO 1 IN EVERY  LQ(I,J)  SUCH THAT CONTOUR  LCO
C    PASSES TO RIGHT OF  (I,J)  BUT NOT TO RIGHT OF  (I+1,J) .
C    SET HIT (LCO+30) TO 1 IN EVERY  LQ(I,J)  SUCH THAT CONTOUR  LCO
C    PASSES ABOVE (I,J) BUT NOT ABOVE (I,J+1) .
  55 DO 66  K = 1, 2
     JS = 3 - K
     KHIT=K
     DO 66  J = JS, N
     DO 66  I = K , M
     DO66LCO=1,NONO
     IF      (CO(LCO).LT.H(I-K+1,J+K-2).AND.CO(LCO).GE.H(I,J)) 64,63
  63 IF      (CO(LCO).GE.H(I-K+1,J+K-2).AND.CO(LCO).LT.H(I,J)) 64,66
  64 WORD=LQ(I-K+1,J+K-2)
     ASSIGN641TONCALL
     GOTONSET1
 641 LQ(I-K+1,J+K-2)=RESULT
  66 CONTINUE
     JJ=1
     II=1
     LOOKHK=0
     GOTO  11
  25 IF (START)  27, 11
  11 I = 0
C    SEARCH FOR UNPLOTTED CONTOUR IN NEIGHBOURHOOD OF  (II,JJ)  .
  51 I = I + 1
     J = 0
  49 DO 43  L0 = 1, 2
     DO 43  L1 = 1, 2
     DO 43  L2 = 1, 2
     IL = II + KL(L1)*KK(L0) * I  +  KL(L1)*KK(L0+1) * J
     JL = JJ + KL(L2)*KK(L0) * J  +  KL(L2)*KK(L0+1) * I
     IF (IL.LT.1. OR .IL.GT.M. OR .JL.LT.1. OR .JL.GT.N)  43, 41
  41 IF(LQ(IL,JL))42,43
  42 IF(LQ(IL,JL).AND.LEFT)44,45
  43 CONTINUE
C    SEARCH UNSUCCESSFUL.  INCREMENT J AND/OR I IF PERMISSIBLE.
     J = J + 1
     IF (J.LE.I)  49, 53
  53 IF (I.LT.M+N)  51, 18
C    NEW CONTOUR(S) FOUND. IDENTIFY THE FIRST ONE.
C    LOCATE A STARTING POINT ON IT AND MOVE THERE, PEN UP.
C    THEN ENTER TRACKING SEQUENCE.
  44 NID = 2
     KS=1
     GOTO  46
  45 NID = 3
     KS=2
  46 II = IL
     JJ = JL
     JS = JL
     START = 1
     KHIT=KS  $  WORD=LQ(II,JS)  $  ASSIGN 470 TO NCALL
     DO 47  LCO = 1, 48
     GOTO NGET
 470 IF(RESULT)22,47
  47 CONTINUE
  22 CALL CONC (NID, II, JJ, CO(LCO), X, Y, M, N, H)
     CALLPLOT(X/XSCALE,Y/YSCALE,3)
     G = 0
     NED = NID
     GOTO  29
C    MAIN PLOTTING SEQUENCE.
C    A POINT HAS BEEN PLOTTED ON ONE SIDE OF A GRID UNIT CELL IN ARRAY  H
C    FIND ANOTHER SIDE OF THIS CELL CUT BY SAME CONTOUR.
  31 START = 0
   1 IF (II.EQ.0.OR.JJ.EQ.0.OR.II.GE.M.OR.JJ.GE.N)  10, 2
   2 DO 54  K = 1, 3
     NSUB=NIDLIST(NID+K)
```

```
      IM=II+ILIST(NSUB)
      JM=JJ+JLIST(NSUB)
      KSIDE(K)=KLIST(NSUB)
   54 LEQ(K)=LQ(IM,JM)
      KBIT=KSIDE(3)   $   WORD=LEQ(3)   $   ASSIGN540TONCALL   $   GOTONGET
  540 IF(RESULT)5,3
    3 KBIT=KSIDE(2)   $   WORD=LEQ(2)   $   ASSIGN300TONCALL   $   GOTONGET
  300 IF(RESULT)9,4
    4 KBIT=KSIDE(1)   $   WORD=LEQ(1)   $   ASSIGN400TONCALL   $   GOTONGET
  400 IF(RESULT)8,10
    5 KBIT=KSIDE(1)   $   WORD=LEQ(1)   $   ASSIGN500TONCALL   $   GOTONGET
  500 IF(RESULT)7,6
    6 KBIT=KSIDE(2)   $   WORD=LEQ(2)   $   ASSIGN600TONCALL   $   GOTONGET
  600 IF(RESULT)25,36
    7 KBIT=KSIDE(2)   $   WORD=LEQ(2)   $   ASSIGN700TONCALL   $   GOTONGET
  700 IF(RESULT)37,25
    9 KBIT=KSIDE(1)   $   WORD=LEQ(1)   $   ASSIGN900TONCALL   $   GOTONGET
  900 IF(RESULT)25,38
   36     NAD = 3
      GOTO  19
   38     NAD = 2
      GOTO  19
   37 CALL  CONC   (NIDLIST(NID+1), II, JJ, CO(LCO), XR, YR, M, N, H)
      CALL  CONC   (NIDLIST(NID+3), II, JJ, CO(LCO), XL, YL, M, N, H)
      IF(XL*XL+YL*YL-XR*XR-YR*YR.LE.2*X*(XL-XR)+2*Y*(YL-YR))   36, 8
    8     NAD = 1
   19 IF(LOOKBK)20,17
   17 NED = NIDLIST (NID + NAD)
C     PLOTTING SEQUENCE ENTRY POINT.
C     TEST COORDINATES OF GRID POINT ON WHICH INTERPOLATION WILL BE BASED.
   29 IT = II + ILIST(NED)
      JT=JJ+JLIST(NED)
      KT=KLIST(NED)
      IF (START)   28, 12
   28 IF ((IT.EQ.1.AND.KT.EQ.2).OR.IT.EQ.M.OR.JT.EQ.1.OR.(JT.EQ.N.AND.KT
     1.EQ.1))  12,15
C     REVERSE DIRECTION OF CONTOUR IF NECESSARY.
   15 LOOKBK=1
      NAD = 0
      II = II + INEW(NID)
      JJ = JJ + JNEW(NID)
      NID = NIDLIST(NID + 2)
      GOTO  1
   20 LOOKBK=0
      II = II + INEW(NID)
      JJ = JJ + JNEW(NID)
      NID = NIDLIST(NID + 2)
      IF (NAD)   31, 12
   12 KBIT=KT   $   WORD=LQ(IT,JT)   $   ASSIGN120 TO NCALL   $   GOTO NSETO
  120 LQ(IT,JT)=RESULT
   30 IF (START)   31, 21
C     INTERPOLATE AND PLOT.
   21     CALL CONC (NED, II, JJ, CO(LCO), X, Y, M, N, H)
      CALLPLOT(X/XSCALE,Y/YSCALE,2)
      G = 1
      NID = NIDLIST(NED + 2)
      II = II + INEW(NED)
      JJ = JJ + JNEW(NED)
      GOTO  31
   10 IF(LOOKBK)20,24
   24 IF (G)   11, 27
   27 II = II + INEW(NID)
      JJ = JJ + JNEW(NID)
      NID = NIDLIST(NID + 2)
      KBIT=KT   $   WORD=LQ(IT,JT)   $   ASSIGN270TONCALL   $   GOTO NSETO
  270 LQ(IT,JT)=RESULT
      GOTO  31
C     PREPARE NEW ORIGIN FOR ANY SUBSEQUENT PLOT
   18 NEWXOR=XMAX+1.96
      CALLPLOT(FLOATF(NEWXOR),0.,-3)
      RETURN
      END

      SUBROUTINE CONC     (NED, I, J, CON, X, Y, MM, NN, H)
      DIMENSIONH(MM,NN)
      IF (NED.EQ.2. OR .NED.EQ.4)   5, 1
    1 K = (NED - 1)/2
      X = I - K
      IF (H(I-K+1,J).EQ.H(I-K+1,J+1))   3, 2
    2 Y = (H(I-K+1,J+1)-CON)/(H(I-K+1,J)-H(I-K+1,J+1)) + J
                                                                 GOTO  7
    3 Y = J - .5
                                                                 GOTO  7
    4 X = I - .5
                                                                 GOTO  7
    5 K = 2 - NED/2
      Y = J - K
      IF (H(I,J-K+1).EQ.H(I+1,J-K+1))   4, 6
    6 X = (H(I+1,J-K+1)-CON)/(H(I,J-K+1)-H(I+1,J-K+1)) + I
    7 RETURN
      END
```

APPENDIX II Antenna polar diagram data

(a) **Christchurch :** Crossed folded dipoles above a ground screen.

Smoothed experimental polar diagram

θ	$G(\theta)$	θ	$G(\theta)$	θ	$G(\theta)$
1	.07	31	1.68	61	3.70
2	.14	32	1.73	62	3.65
3	.23	33	1.80	63	3.58
4	.33	34	1.88	64	3.50
5	.43	35	1.98	65	3.41
6	.54	36	2.09	66	3.32
7	.65	37	2.20	67	3.22
8	.76	38	2.32	68	3.12
9	.89	39	2.43	69	3.02
10	1.01	40	2.55	70	2.92
11	1.12	41	2.67	71	2.82
12	1.22	42	2.81	72	2.72
13	1.30	43	2.96	73	2.62
14	1.37	44	3.10	74	2.52
15	1.42	45	3.23	75	2.43
16	1.46	46	3.35	76	2.34
17	1.49	47	3.45	77	2.26
18	1.51	48	3.54	78	2.19
19	1.52	49	3.62	79	2.12
20	1.52	50	3.68	80	2.06
21	1.52	51	3.73	81	2.01
22	1.52	52	3.77	82	1.97
23	1.52	53	3.79	83	1.93
24	1.51	54	3.81	84	1.90
25	1.51	55	3.81	85	1.87
26	1.52	56	3.81	86	1.85
27	1.54	57	3.80	87	1.83
28	1.56	58	3.79	88	1.82
29	1.60	59	3.77	89	1.82
30	1.63	60	3.74	90	1.81

(b) **Ottawa :** Crossed dipoles 0.4λ above a ground screen.

Theoretical polar diagram

$$G(\theta) = \left[\frac{\cos^2(\frac{\pi}{2} \cos \theta)}{\sin^2 \theta} + 1 \right] \sin^2 (0.8\pi \sin \theta)$$

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Elford W. G. Calculation of the response function of the Harvard Radio Meteor Project radar system. Harvard-Smithsonian Radio Meteor Project Res. Report. No. 8. (1964).

Elford W. G. The incidence of meteors on the earth derived from radio observations. Accepted for publication in the Smithsonian Contributions to Astrophysics.

McIntosh B. A. The determination of meteor mass distribution from radar echo counts. Can. J. Physics. 44, 2729 (1966).

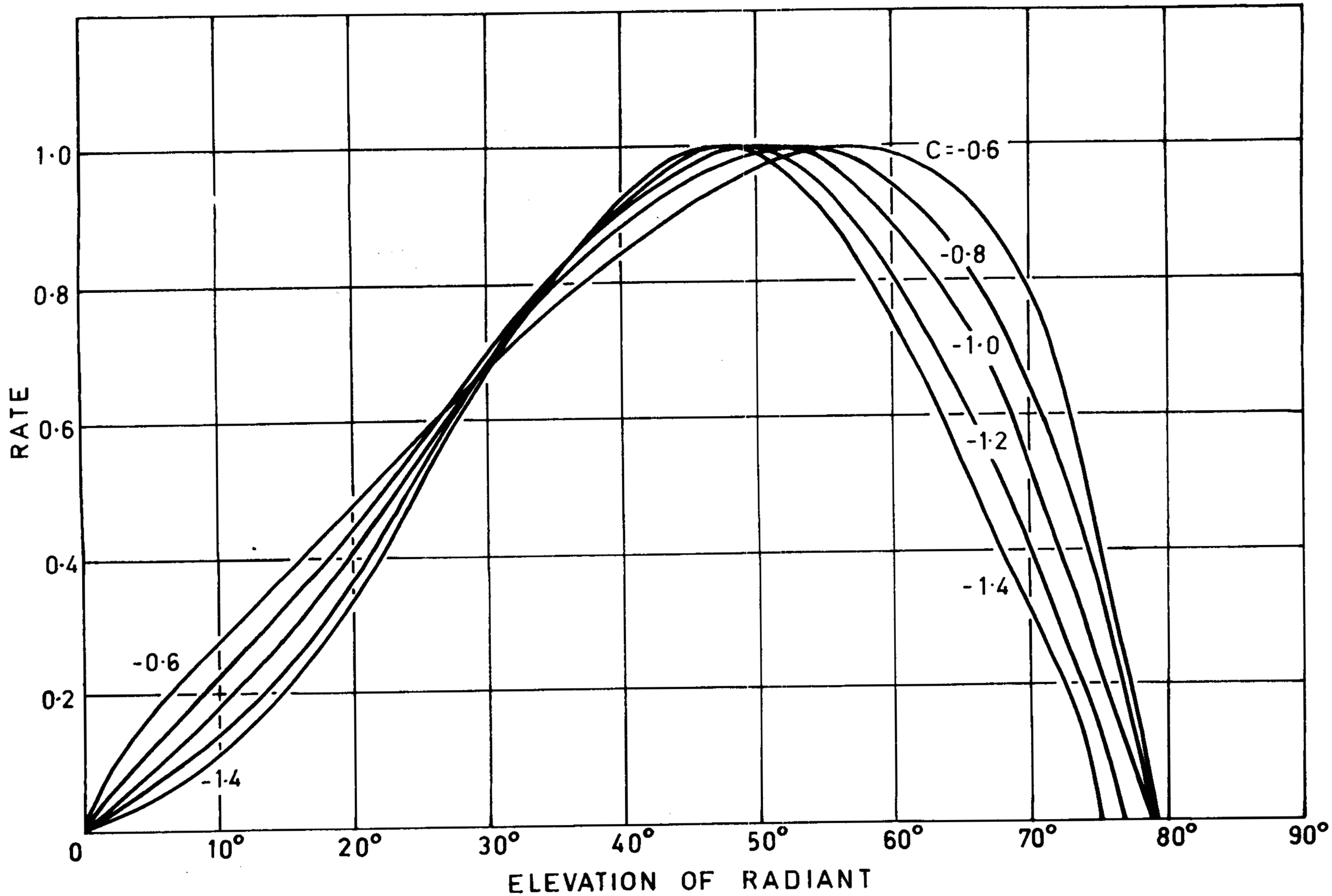


__Figure 1.__ Response functions for the 33 mc/s system at Ottawa, calculated for several values of the exponent C in the distribution law. Meteors detected between the range limits 100–360 km.

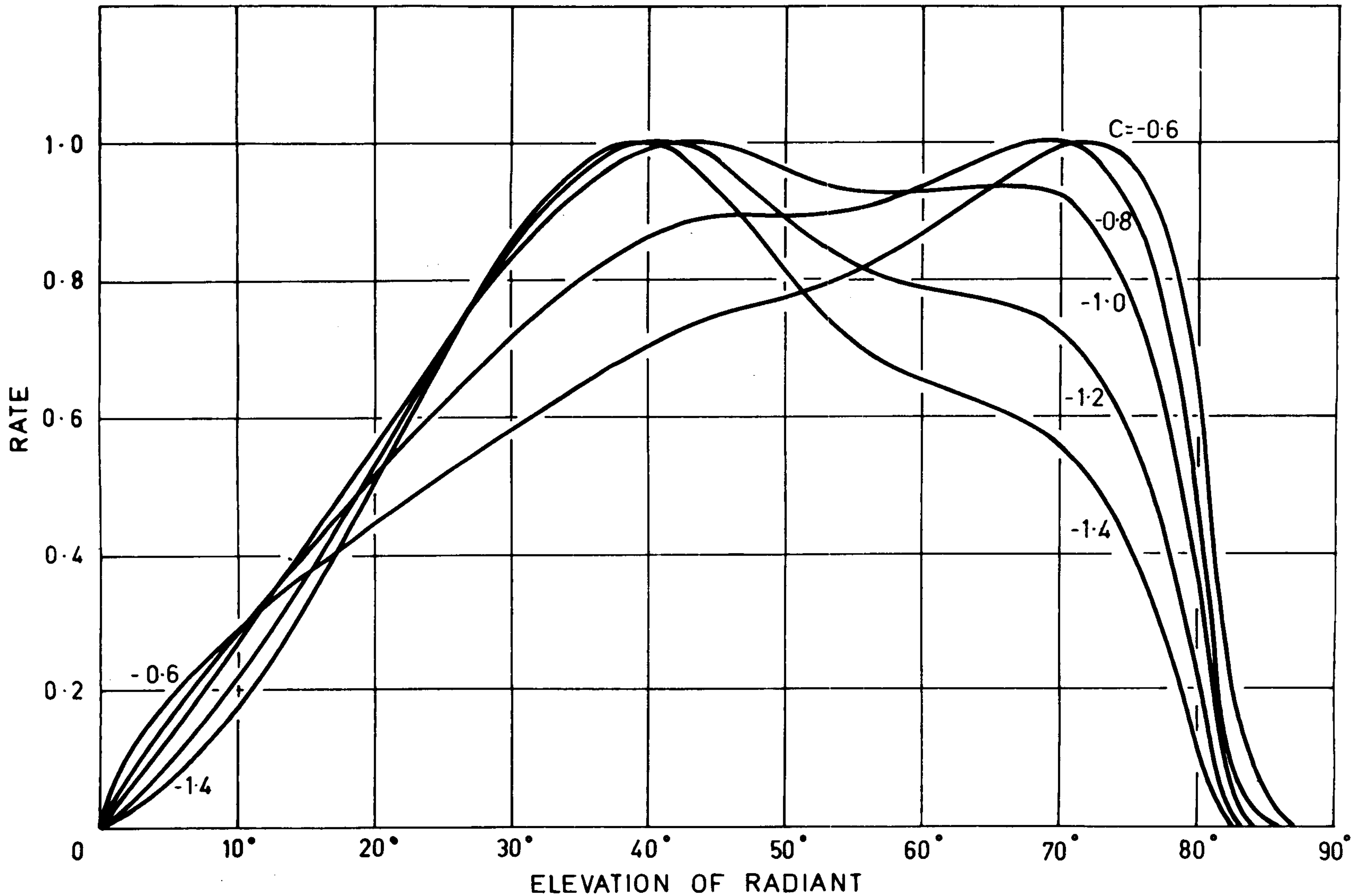
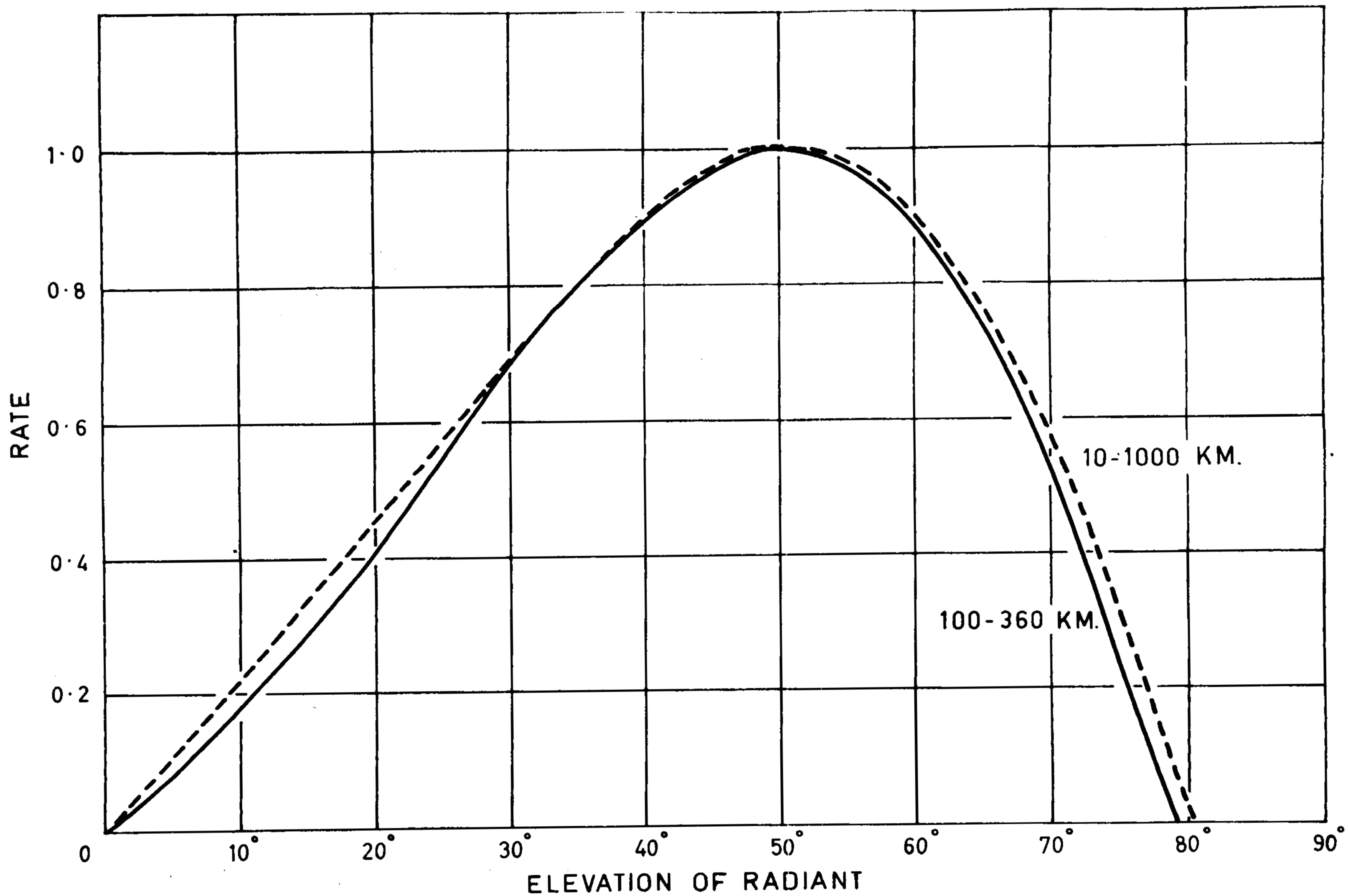


Figure 2. Response functions for the 70 mc/s system at Christchurch, calculated for several values of the exponent C in the distribution law. Meteors detected to a maximum range of 1,000 km.



<u>Figure 3</u>. Normalized Ottawa response functions for two range limits (C = -1.0).

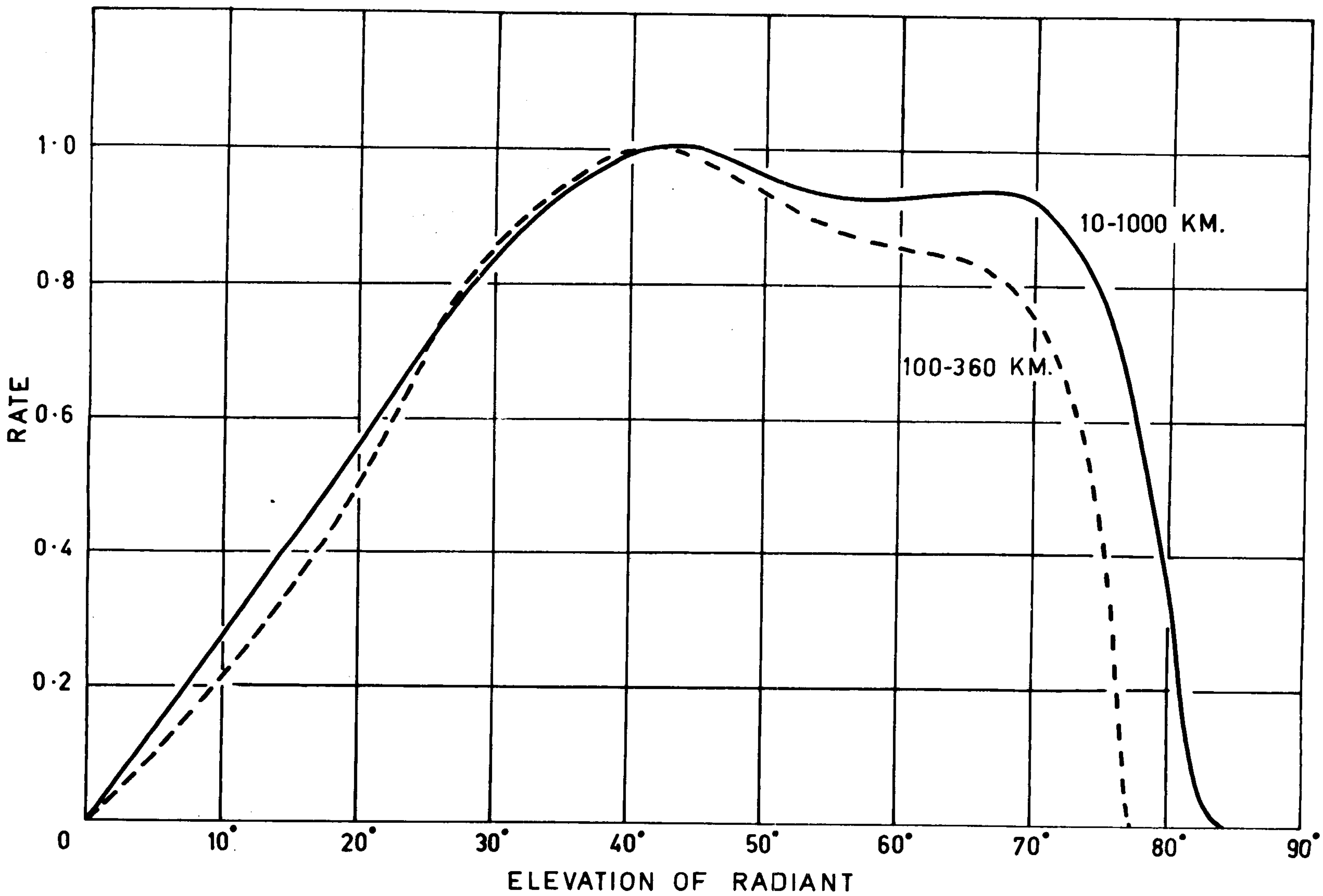


Figure 4. Normalized Christchurch response functions for two range limits (C = -1.0).

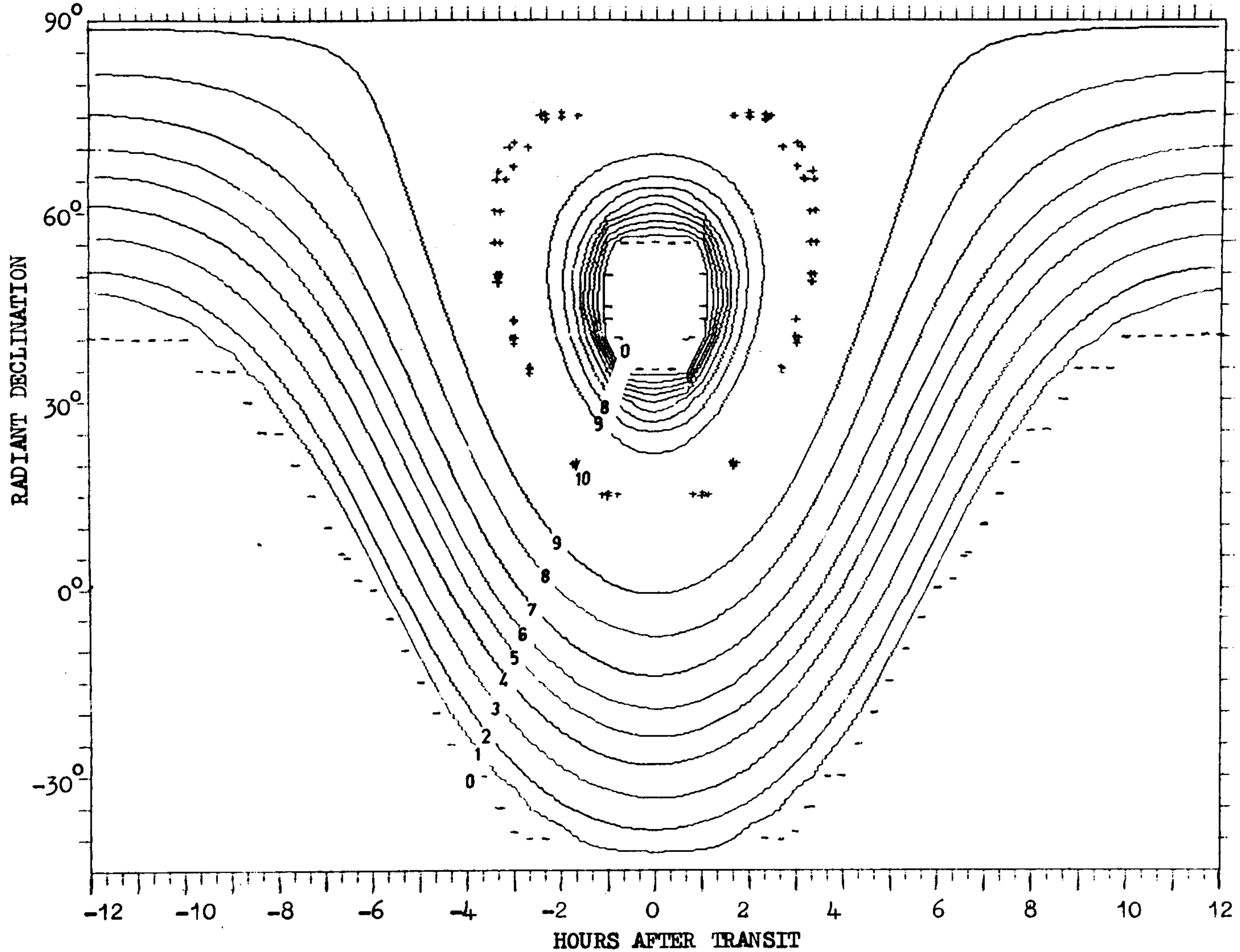


Figure 5

Echo rate as a function of radiant declination and time for the Ottawa system, C = -0.6

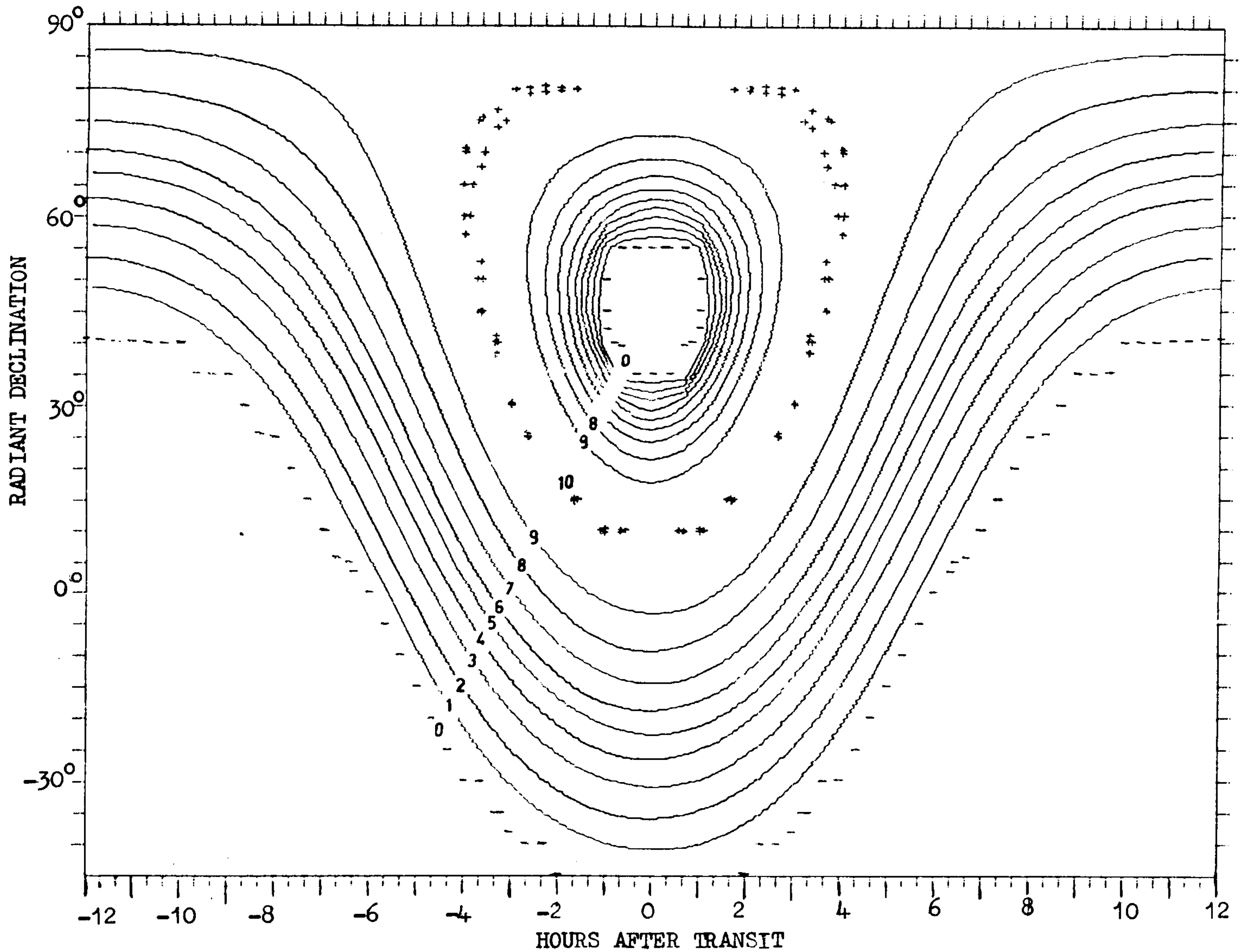


Figure 6

Echo rate as a function of radiant declination and time for Ottawa system, C = -.8

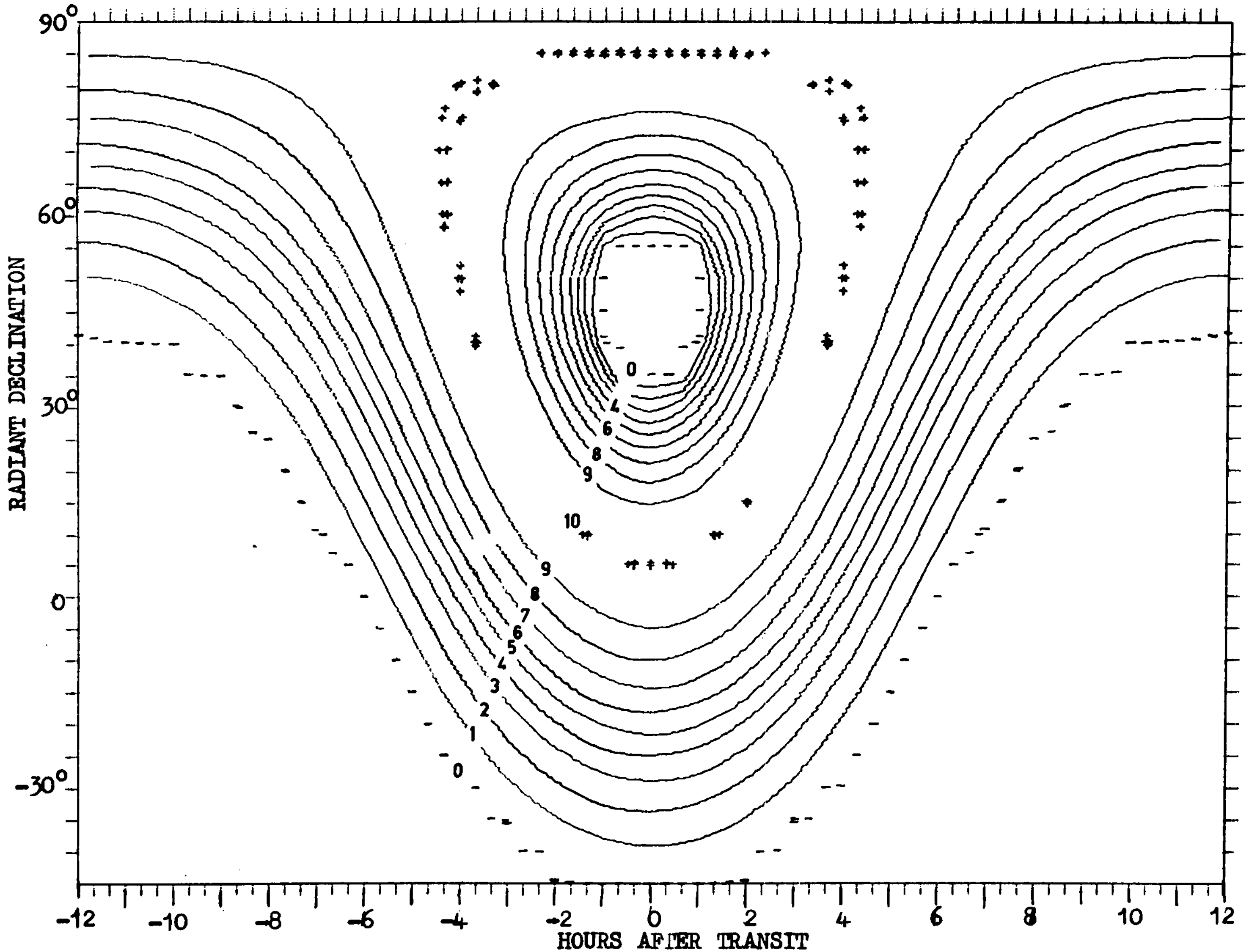


Figure 7

Echo rate as a function of radiant declination and time for the Ottawa system, C = -1.0

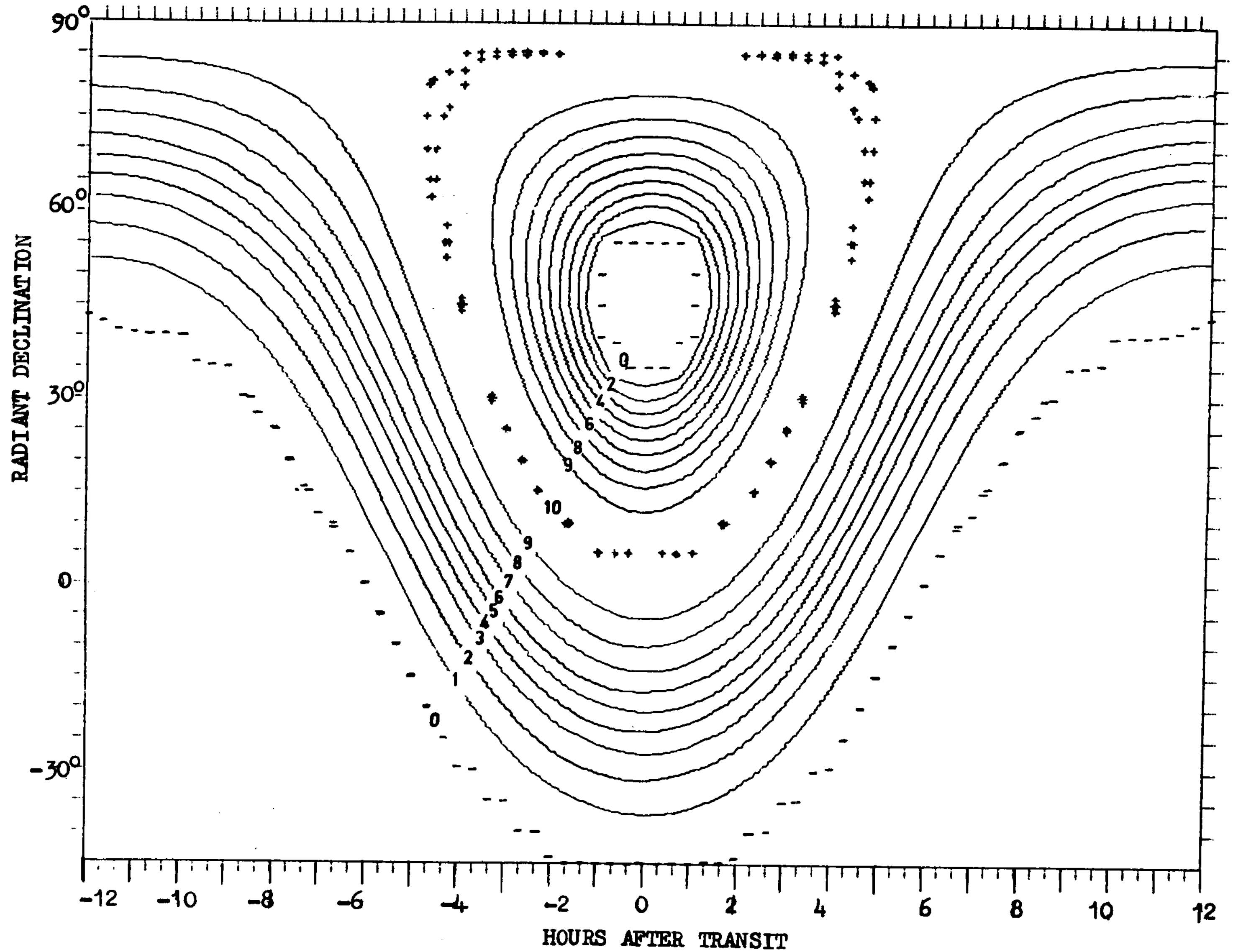


Figure 8.

Echo rate as a function of radiant declination and time for the Ottawa system, C = -1.2

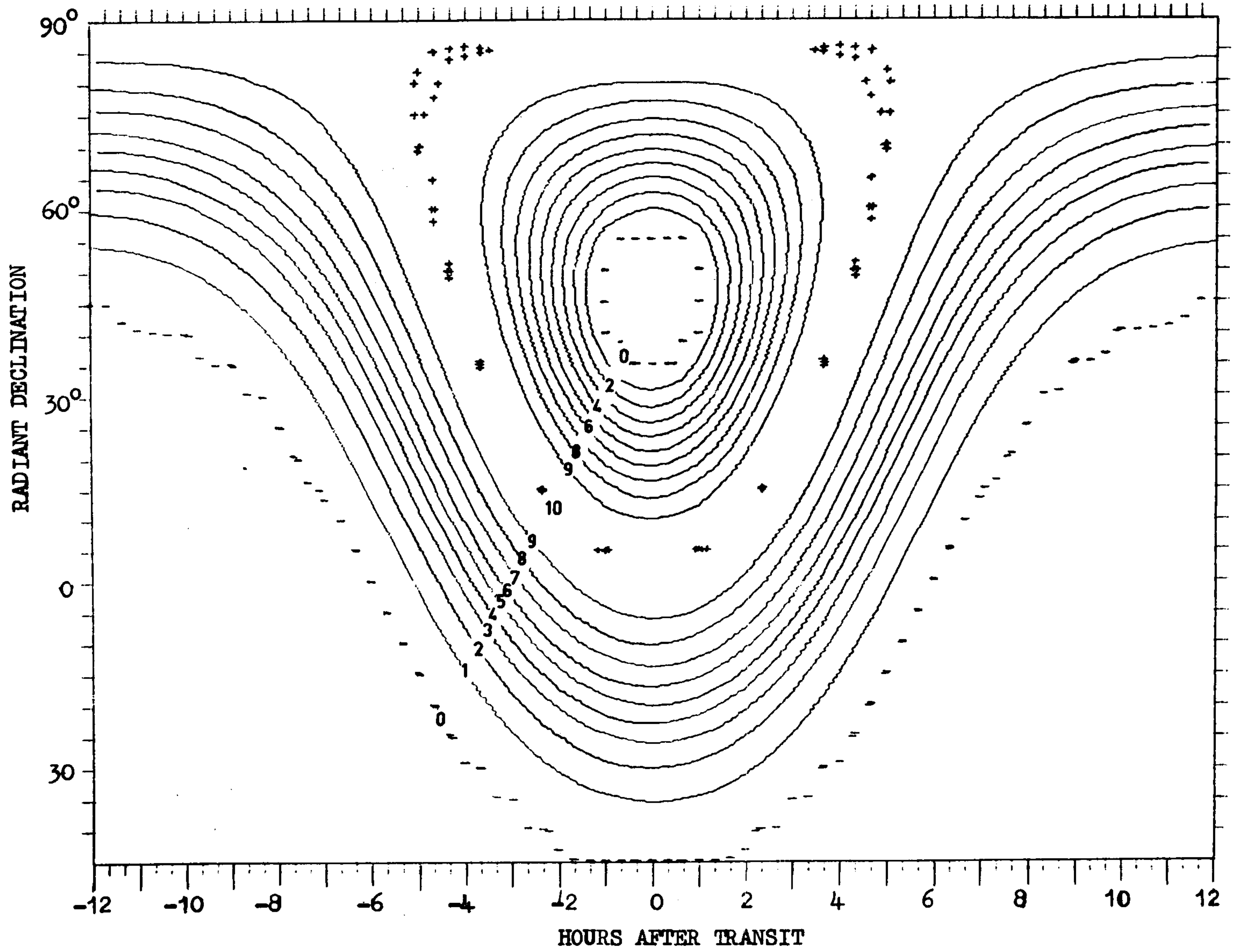
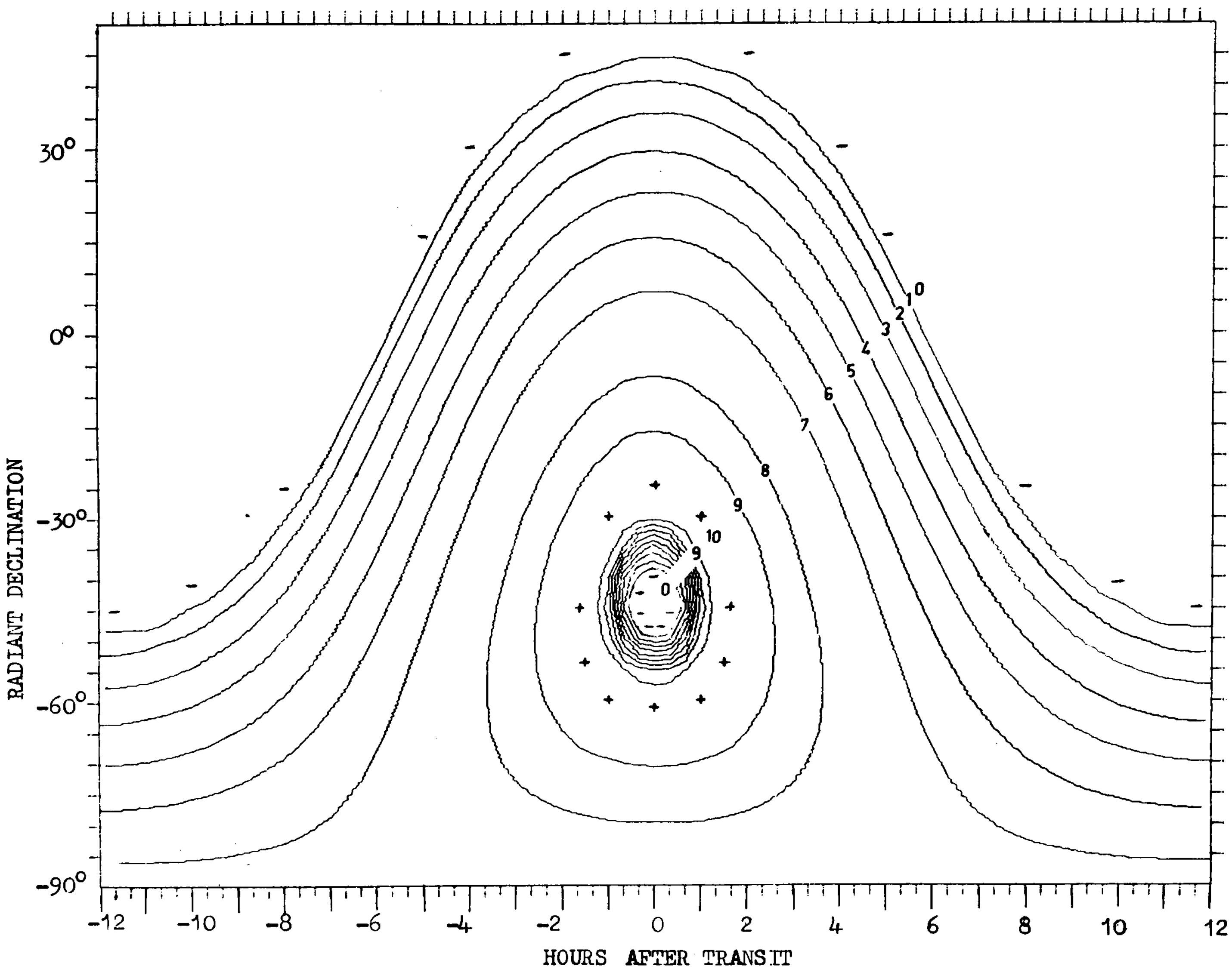


Figure 9

Echo rate as a function of radiant declination and time for the Ottawa system, C = -1.4



<u>Figure 10</u>

Echo rate as a function of radiant declination and time for the Christchurch system, C = -0.6

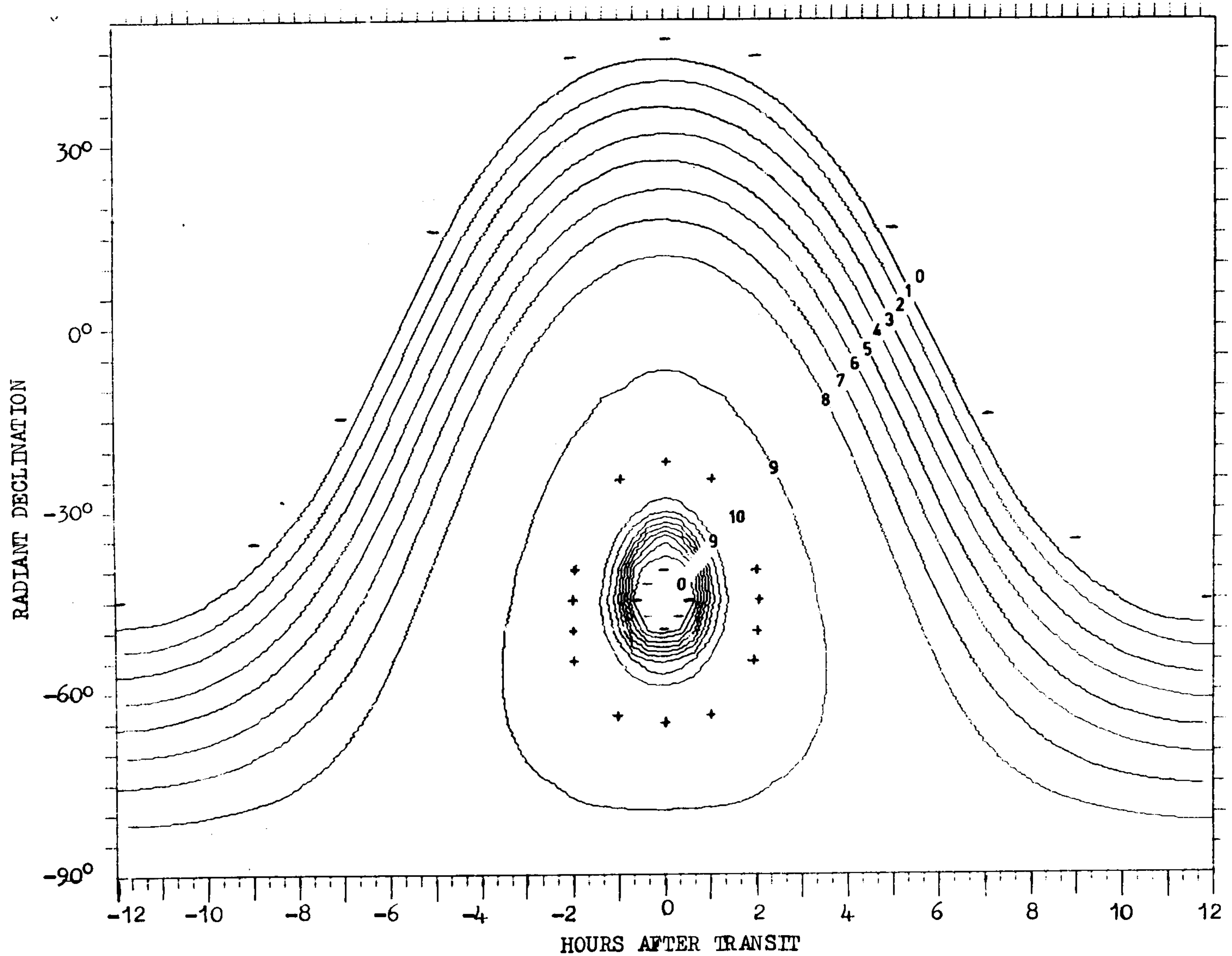


Figure 11

Echo rate as a function of radiant declination and time for the Christchurch system, C=-.8

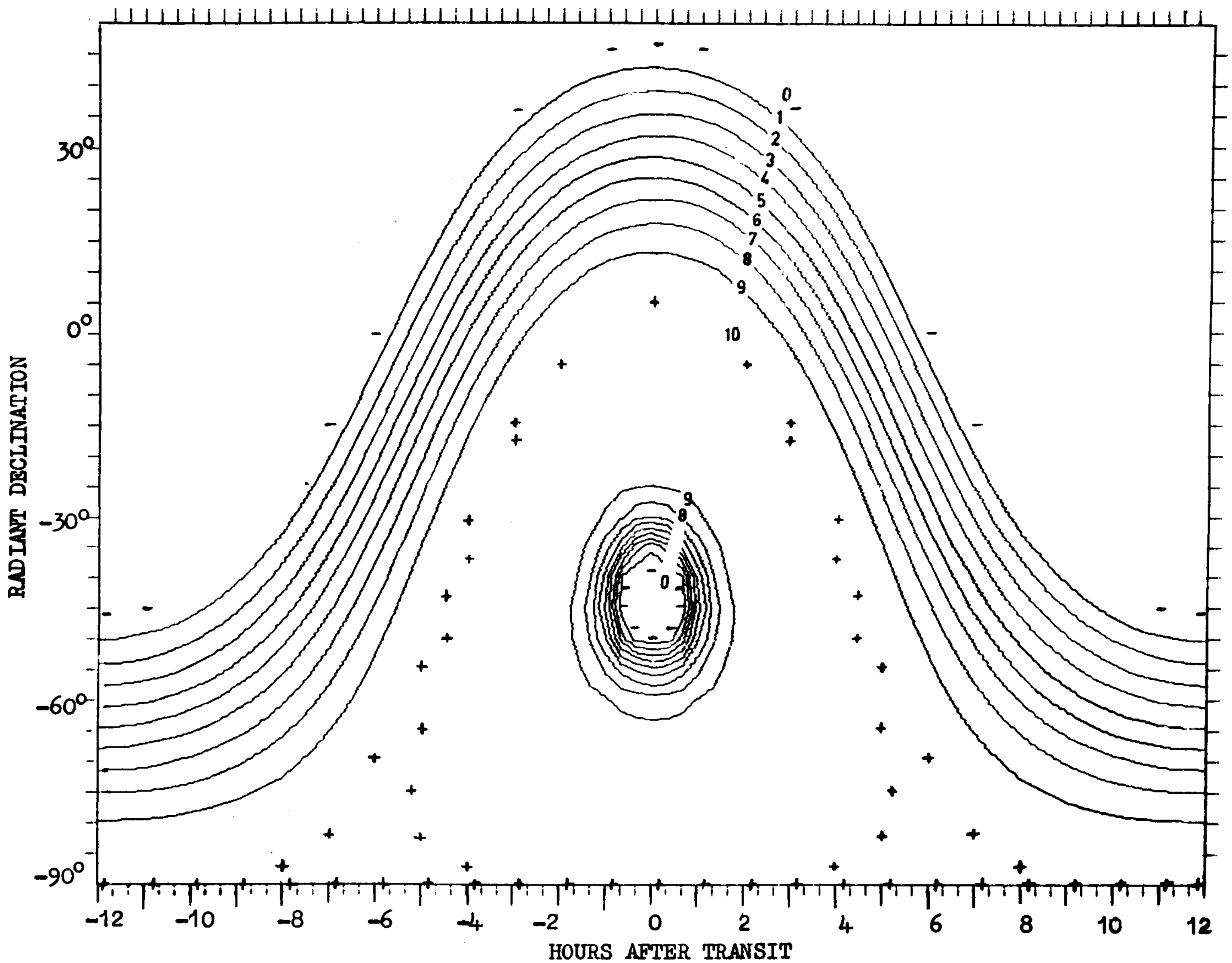


Figure 12

Echo rate as a function of radiant declination and time for the Christchurch system, C=-1.0

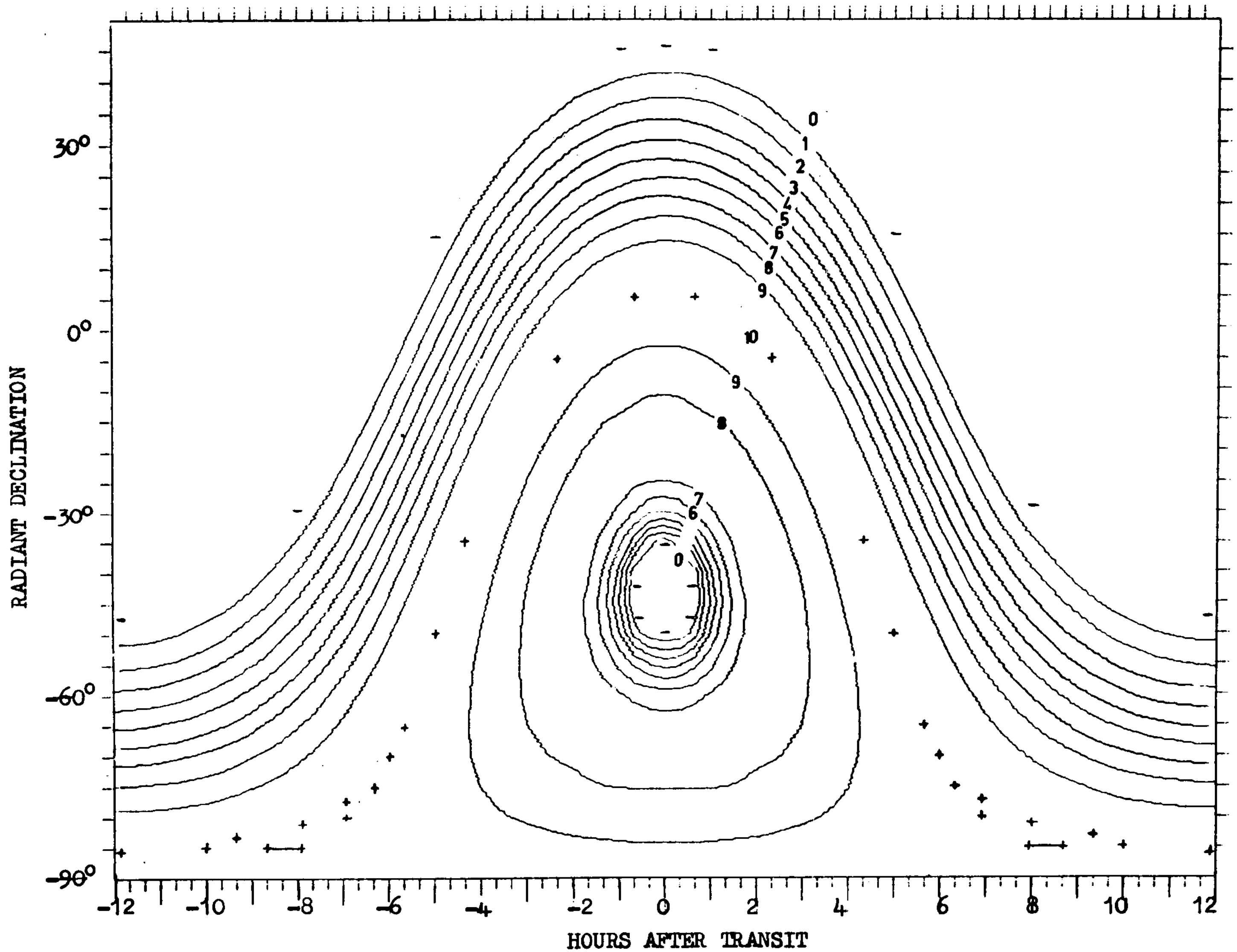


Figure 13

Echo rate as a function of radiant declination and time for the Christchurch system, C = -1.2

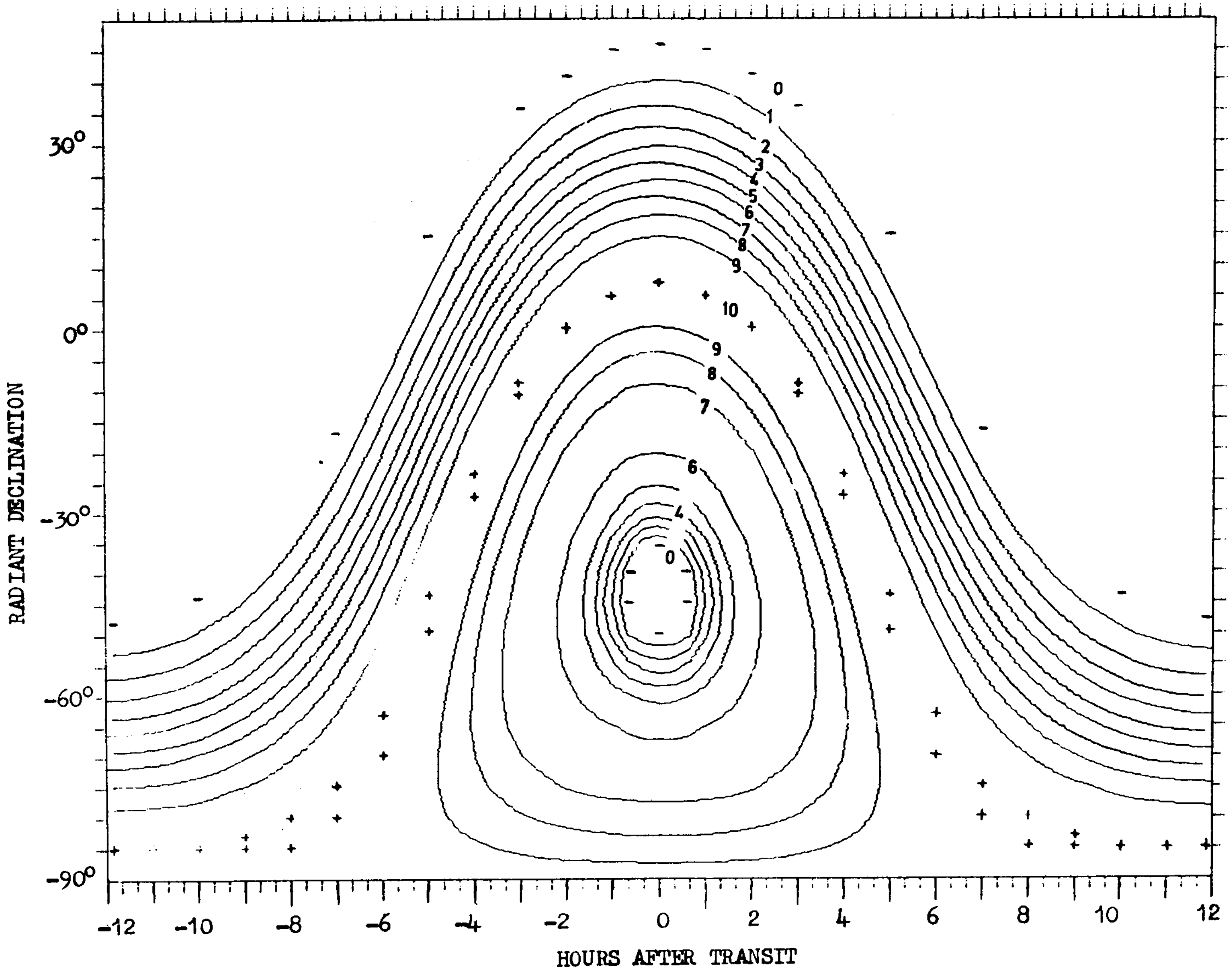


Figure 14

Echo rate as a function of radiant declination and time for the Christchurch system, C = -1.4

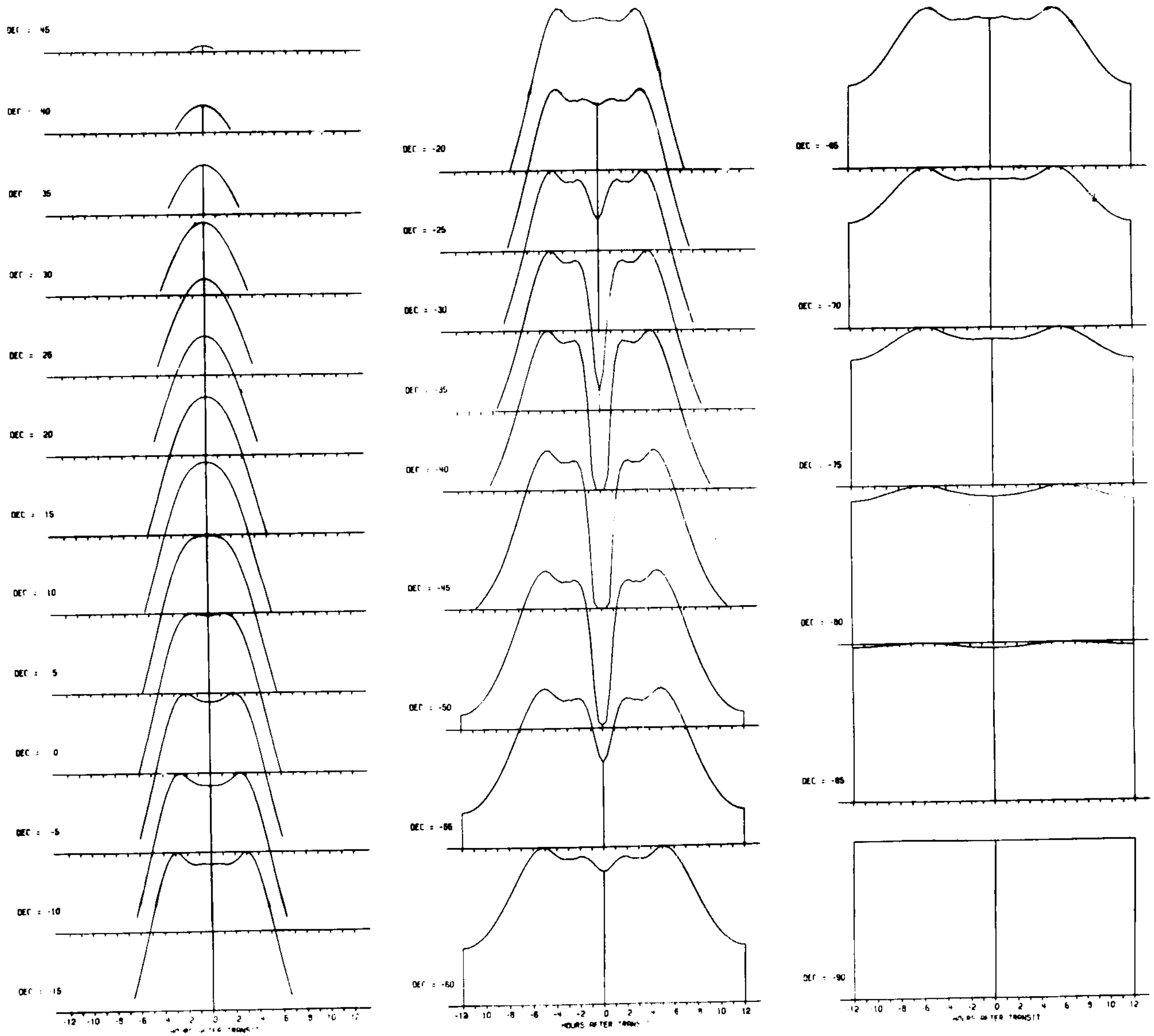


Figure 15. Diurnal rate curves as a function of radiant declination (5° intervals, C = -1.0): Christchurch.

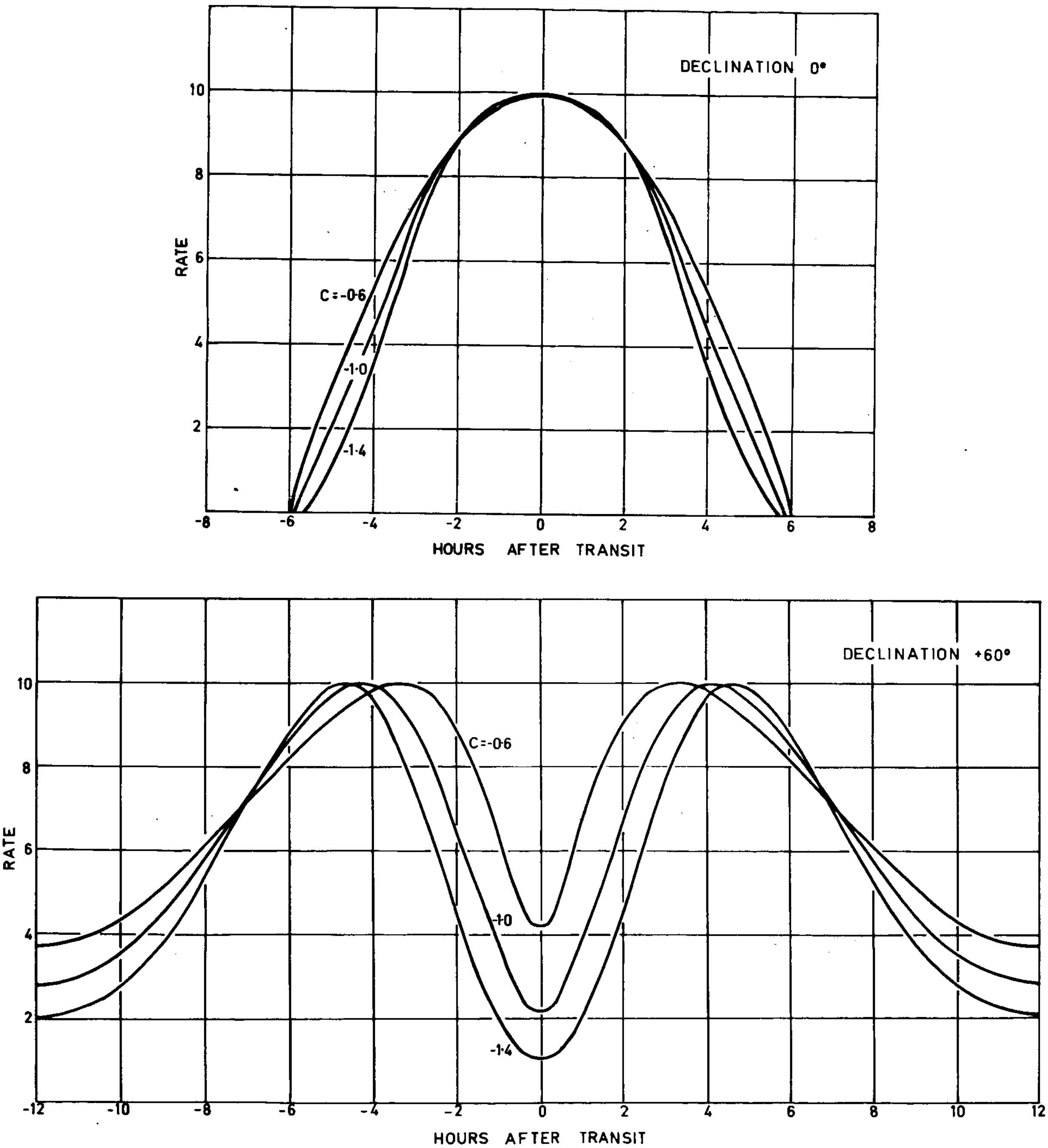


Figure 16. Ottawa diurnal rate curves for radiants at declinations 0°, + 60° and for $\overline{C}$ = −0.6, −1.0, −1.4. Curves have been normalized to equal maxima.

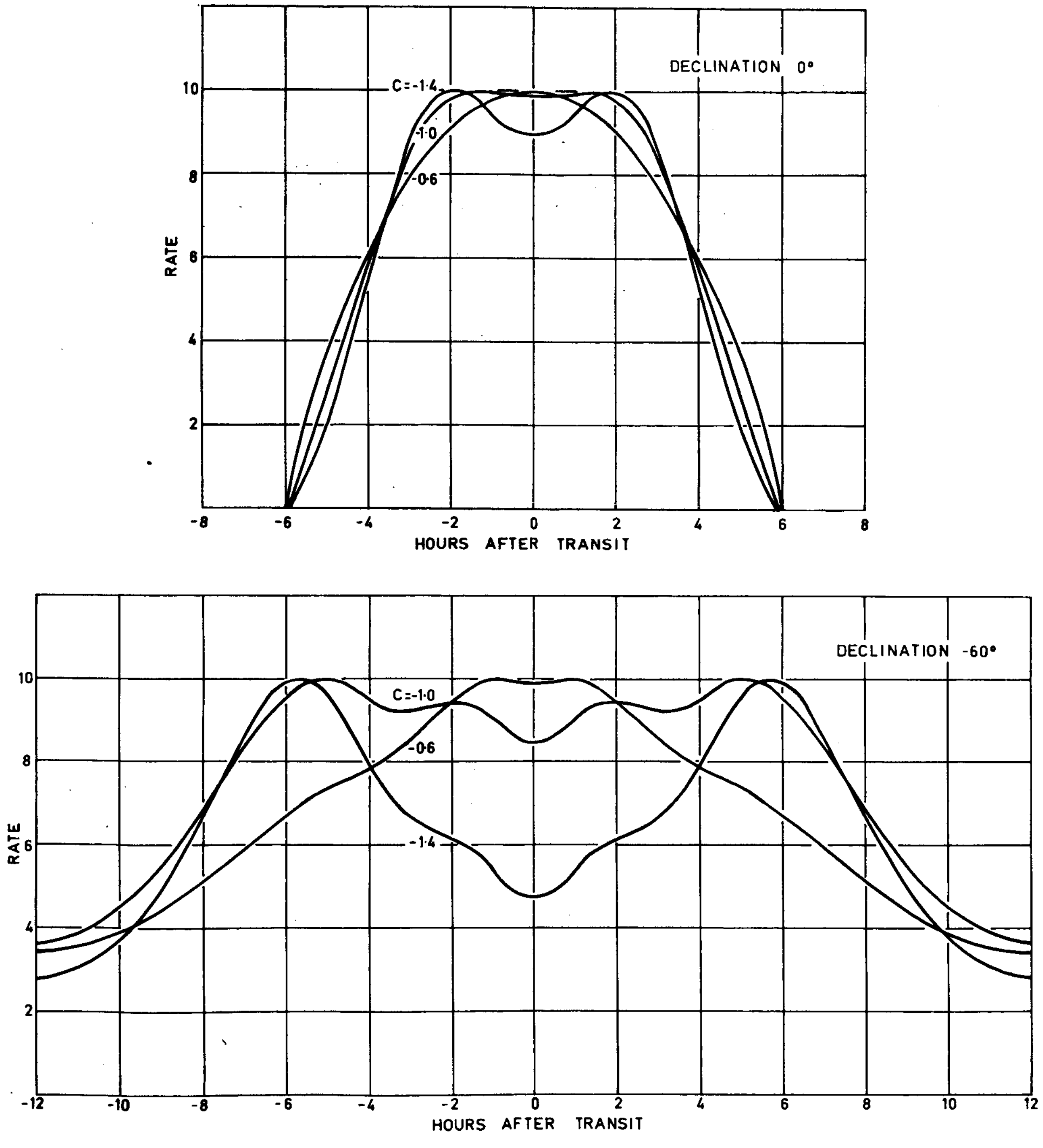


Figure 17. Christchurch diurnal rate curves for radiants at declinations 0°,-60° and for C = -0.6, -1.0, -1.4. Curves have been normalized to equal maxima.